M1 General Physics

Particles

Relativistic kinematics

Exercise 1: Relative velocity

Consider two particles of momenta p^μ and q^μ. Provide a covariant expression of their relative velocity, defined in a frame in which one of the two is at rest. It will be useful to compute $p \cdot q$ in such a frame.

Exercise 2: Fixed target experiments versus collider experiments

1. We send a particle of mass m, of kinetic energy K (total energy minus energy at rest, i.e $K = E - m$), on another identical particle at rest (so-called *fixed target* experiment). Compute the energy in the center-of-mass frame.

2. We accelerate two particles of opposite momenta, of mass m and of kinetic energy K^* (collider mode, the symbol $*$ is used to refer to the center-of-mass frame). What should be the value of K in the fixed target experiment of question 1. in order to reach the same energy in the center-of-mass frame? Discuss the limits $K^* \ll m$ and $K^* \gg m$. Explain why most of the modern experiments in high-energy physics are made using colliders (LEP at CERN, HERA at Hamburg, Tevatron at Fermilab near Chicago, SLC in Stanford, LHC, future EIC (2030), ILC e^+e^- projects, FCC-ee and FCC-pp).
The RHIC collider, at Brookhaven (near New-York) collides two beams of heavy ions, for example Au-Au with 200 GeV per nucleon. Compute the energy K of a similar fixed target experiment. One should compare this result to the highest possible energy available at SPS, which was a fixed target experiment (beams of 160 GeV per nucleon).

3. One collides two beams of ultra-relativistic particles ($E \gg m$) in the opposite direction, of energies E_1 and E_2. Compute the center-of-mass energy.
The HERA collider (1992-2007) was colliding a beam of protons of 800 GeV with a beam of electrons of 30 GeV. Compute the center-of-mass energy.

Exercise 3: Photoproduction of pions

Consider the reaction $\gamma p \to \pi^0 p$, where p means a proton, of mass $M = 939$ MeV, γ is a photon and π^0 a neutral pion, of mass $m \simeq 135$ MeV. We denote respectively P, $k = (k^0, \vec{k})$ and $q = (q^0, \vec{q})$ the four-momenta of the incoming proton, of the incoming photon and of the emitted pion.

1. We assume that the proton is initially at rest. Compute, literally and then numerically, the minimal energy of the photon in order that the reaction $\gamma p \to \pi^0 p$ would be possible.

2. Compute the energy of the incident photon as a function of the emitted pion energy q^0 and of the angle θ of the pion momenta with respect to the incident photon.

3. Simplify the previous results in the limit where m and k^0 are very small with respect to M. Comment on the result.

4. This reaction plays a crucial role in the physics of high energy cosmic rays: indeed, the Universe is bathed with photon (the cosmological radiation at 3K) and protons of very high energy can scatter with these photons and produce pions, thus slowing them down. Consider thus the scattering of a proton of energy E and a photon of energy $k^0 = 10^{-3}$ eV of opposite direction (this is the order of magnitude of the 3K radiation). Compute the minimal value of E in order that the reaction would be possible. This cut is known under the name of Greisen-Zatsepin-Kuzmin (1966). One of the enigma of current studies of cosmic rays is the fact that there are signals of cosmic rays of higher energies.

Exercise 4: Compton effect

Compton scattering is the elastic scattering of a photon of momentum $\vec{k}$ with an electron of mass m and of momentum $\vec{p}$. We denote by $\vec{k}'$ the momentum of the scattered photon, $E = \sqrt{\vec{p}^2 + m^2}$ the energy of the incoming electron, and θ the angle between $\vec{k}$ and $\vec{p}$.

1. Check that $P \cdot K = (P + K) \cdot K'$.
Infer that the energy of the scattered photon equals

$$|\vec{k}'| = \frac{E - p\cos\theta}{E + |\vec{k}| - \hat{\vec{k}}' \cdot (\vec{p} + \vec{k})} |\vec{k}| \tag{1}$$

where we denote as $\hat{k}' \equiv \vec{k}'/|\vec{k}'|$ the direction of $\vec{k}'$.

2. We assume that the electron is at rest. Compute the wave length λ' of the scattered photon as a function of the wave length λ of the incoming photon, of the Compton wave length defined as $\lambda_C = h/mc$, and of the angle θ' between $\vec{k}$ and $\vec{k}'$.

3. In the more general case where $\vec{p} \neq \vec{0}$, deduce from equation (1) the maximal energy of the diffused photon for a given θ.

4. We assume the electron to be ultrarelativistic. Check that in the limit of a low energy photon,

$$|\vec{k}'|_{\max} = \left(\frac{2E\sin(\theta/2)}{m}\right)^2 |\vec{k}| \,. \tag{2}$$

Specify the condition on $|\vec{k}|$ for this equation to be valid. Check the validity of the various approximations and calculate $|\vec{k}'|_{\max}$ in the case of a laser beam of energy 1 eV interacting with an electron beam of energy $E = 6$ GeV, 12 GeV since 2016, in opposite direction. These are typical value of the Compton polarimeter used at Thomas Jefferson National Accelerator Facility (JLab), located in Virginia (USA). Such a device allows, using a polarized laser, to measure the polarization of the electron beam. It also allows to produce high energy photons with a partial polarization (using the maximal energy of the produced photon corresponding to the backscattering regime).

Exercise 5: Electron scattering

In this exercise, we consider the scattering, either elastic or inelastic, of an ultrarelativistic electron (its mass will be neglected) on a target. We denote by $K = (k, \vec{k})$ and $K' = (k', \vec{k'})$ the four-momenta of the electron before and after the collision, and $P = (E, \vec{p})$ and P' those of the target, which has a mass M. We assume that $\vec{k}$ and $\vec{p}$ are collinear and of opposite directions.
If the scattering is elastic, the energy k' of the outgoing electron is a function of the angle between $\vec{k}$ and $\vec{k'}$ (see the previous exercise). This angle will be denoted here by θ. In the more general case of an inelastic scattering, k' and θ are independent variables.

1. It turns out to be convenient to use, instead of k' and θ, the Lorentz invariant variables Q^2 and $P \cdot q$, where $q \equiv K - K'$. Compute these two quantities as functions of k', θ, E and k. What is the sign of q^2? Traditionally, one denotes $Q^2 = -q^2$.

2. Consider a scattering which transforms the target into a particle of mass M', with $M' \geqslant M$. What is the relationship between Q^2 and $P \cdot q$? Infer the sign of $P \cdot q$. Whenever M' is close to M, such a scattering is named quasi-elastic.

3. We assume that the electron faces an elastic scattering off a part of a target, characterized by a four-momentum xP, with $0 \leq x \leq 1$, the rest of the target being spectator, not involved in the collision. Express x as a function of Q^2 and $P \cdot q$. Such a scattering is named deep inelastic (DIS) when Q^2 is large (with respect to Λ^2_{QCD}). In the case of DIS at high energy (a few GeV or more) on a proton, x can be interpreted as the momentum fraction carried by a quark (constituent of the proton) scattered by the incoming electron.

4. Draw, in the Q^2, $P \cdot q$ plane, the lines corresponding to DIS at fixed x, as well as the lines corresponding to quasi-elastic scattering.

5. The target is assumed to be at rest. Draw, again in the Q^2, $P \cdot q$ plane, the lines corresponding to fixed k' as well as those corresponding to fixed θ. Deduce from that the allowed kinematical region. Compute the maximal value of Q^2 for a given x, in the limit $k \gg M$.

6. Consider again the previous question, in the case of a collider, in which the target moves at an ultra-relativistic energy $E \gg M$. In the case of HERA (DESY, Hamburg, 1992-2007), $k = 30$ GeV and the target was a proton of energy $E = 800$ GeV. Compute the maximal value of Q^2 for $x = 10^{-4}$. Why is it interesting to accelerate protons? In the future (circa 2030), EIC will scatter electron beams on proton and ions beams.