On the Nucleon-Antinucleon Interactions (*).

M. GOURDIN, B. JANCOVICI and L. VERLET

Laboratoire de Physique de l'Ecole Normale Supérieure - Université de Paris

(ricevuto il 25 Marzo 1958)

Résumé. — Les sections efficaces nucléon-antinucléon ont été calculées à 167 MeV avec deux modèles à puits de potentiel complexe. On a d'abord étudié des puits complexes de Yukawa, dans l'approximation de Born. Un potentiel semi-théorique a ensuite été analysé sur l'Ordinateur électronique IBM 704: la partie réelle est le potentiel de Signell et Marshak, adapté au problème nucléon-antinucléon, et la partie imaginaire, un puits phénoménologique de Yukawa. Pour rendre compte des résultats expérimentaux (grande section efficace totale et petite section efficace de diffusion), il apparaît nécessaire que le potentiel réel soit faible et que le potentiel imaginaire ait une portée au moins égale à la portée usuelle des forces nucléaires. La théorie mésique de la source fixe d'une part conduit à un potentiel réel trop fort, et d'autre part ne prédit pas de potentiel imaginaire; cette théorie ne peut pas rendre compte des résultats expérimentaux.

1. – Introduction.

The experiments (1) on the nucleon-antinucleon cross-sections, although they are still in a preliminary stage, exhibit two striking features which deserve theoretical understanding:

1) For a given energy, the total cross-sections are much larger than in the nucleon-nucleon case: at 190 MeV in the laboratory system, the anti-

(*) Supported in part by the United States Air Force through the European Office Air Research and Development Command.

(1) O. CHAMBERLAIN, E. SEGRÈ, C. WIEGAND and T. YPSILANTIS: *Phys. Rev.*, **100**, 947 (1955); B. CORK, G. R. LAMBERTSON, O. PICCIONI and W. A. WENTZEL: *Phys. Rev.*, **104**, 1193 (1956); **107**, 248 (1957); O. CHAMBERLAIN, D. V. KELLER, R. MERMOD, E. SEGRÈ, H. H. STEINER and T. YPSILANTIS: *Phys. Rev.*, **108**, 1553 (1957).

proton-proton cross-section is of the order of 135 millibarns, in contrast with the 22 mb obtained with protons of the same energy.

2) The total cross-section seems to be essentially due to annihilation. If we choose to believe the experiments in their present state, there seems to be little room for elastic and exchange scattering, which contribute probably less than 20% of the total cross-section [2].

The large total cross-section has been explained by BALL and CHEW [3] using a model inspired by the success of the fixed source meson theory. This theory leads, in the nucleon-nucleon case, to the Gartenhaus potential [4]; when a phenomenological spin-orbit term is added [5], it gives a good fit to the scattering and polarization data up to 150 MeV. A nucleon-antinucleon interaction can be obtained by reversing the sign of the second order term of the Gartenhaus potential, to take into account the charge conjugation between nucleon and antinucleon. BALL and CHEW make the two further assumptions:

1) The Signell-Marshak spin-orbit term remains unchanged when one goes to the nucleon-antinucleon case.

2) The inner region of the interaction ($r < 0.4(\hbar/\mu c)$, where μ is the π-meson mass), of which little can be said from meson theory, is replaced by an absorbing core in which the annihilation takes place. The total cross-section at 140 MeV, computed in W.K.B. approximation with such an interaction is in good agreement with experiment. The reason why the total cross-section is larger for antinucleons than for nucleons seems to be that, in the latter case, there is a cancellation between 2nd and 4th order terms. This cancellation disappears in the antinucleon problem where the sign of the 2nd order term has been changed.

The small elastic and exchange cross-sections are much more difficult to explain. The absorption predicted by CHEW and BALL is quite small (69 mb). As we shall see later, an exact calculation with the same model leads to even smaller results. This is in definite contradiction with the experiments if we choose to believe them in their present state.

Similar results were obtained by KOBA and TAKEDA [6], who also considered a small black sphere, surrounded by a real square well.

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[2] O. CHAMBERLAIN: private communication.

[3] J. S. BALL and G. F. CHEW: UCRL-3922 (1957); *Phys. Rev.*, **109**, 1385 (1958).

[4] S. GARTENHAUS: *Phys. Rev.*, **100**, 900 (1955).

[5] P. S. SIGNELL and R. E. MARSHAK: *Phys. Rev.*, **106**, 832 (1957).

[6] Z. KOBA and G. TAKEDA: to be published.

Actually, the failure of the above models to account for a small scattering cross-section is not surprising, and should be related to the choice of ranges which are too small, especially as far as the absorbing region is concerned. It has been shown, in fact, by very general arguments (7) that, in order to obtain a small scattering cross-section together with a large total cross-section, it is necessary to have many partial waves, and thus a large extension of the absorbing potential.

We shall therefore introduce, in a phenomenological way, a long range imaginary potential. Some theoretical argument has been put forward by M. LÉVY (8) in favour of such a potential. In a preliminary exploration (Sect. **2**), we investigate the scattering and annihilation cross-sections obtained with simple real and imaginary Yukawa potentials, having independent depths and ranges. Agreement with experiment requires that the imaginary potential has a range at least of the order of the meson Compton wave length, and that the real potential has a relatively small depth.

In a following Section (**3**), we try to improve Ball and Chew's model by adding to their real potential a phenomenological Yukawa shaped imaginary potential. An exact phase-shift analysis leads us to the conclusion that the fixed source meson theory in its present state provides a real potential which is much too strong to account for the small scattering cross-sections.

2. – Complex potential with Yukawa wells.

In this Section, we intend to calculate in Born approximation, the scattering and absorption cross-sections with the complex potential:

$$V(r) = -V_R \frac{\exp[-\alpha r]}{\alpha r} - iV_I \frac{\exp[-\beta r]}{\beta r}\,.$$

The choice of a Yukawa shape, at least for the imaginary part, is related to the physical assumption that the absorption should be strong at short distances. The Yukawa shape has also the mathematical advantage that the Born approximation can be calculated analytically up to the second order (9); the necessary formulae are given in Appendix I. It can be seen that, with our choice of parameters, the convergence of the Born series is good. For a fixed total cross-section, the elastic scattering increases when the ranges α^{-1} and

(7) M. LÉVY: *Padua-Venice Conference* (1957); W. RARITA: *Bull. Amer. Phys. Soc.*, **2**, 354 (1957).

(8) M. LÉVY: to be published in *Nuovo Cimento*.

(9) P. MORSE and H. FESHBACH: *Methods of Theoretical Physics* (New York, 1953).

β^{-1} decrease. This result confirms the need for long range potentials in order to fit the small elastic to total cross-section ratio.

We proceed to a more quantitative study for the incident wave number $k = 2\mu$. This wave number corresponds to an incident antiproton energy of 167 MeV in the laboratory system. We consider several sets of values for the ranges α^{-1} and β^{-1}, and, of each of them, first choose $V_R = 0$. V_I is adjusted so as to yield 120 mb for the corresponding total cross-section. We then vary V_R and look for the maximum value of V_R which is allowed if there is to be no more than 30 mb for the elastic cross-section. The results are in Table I.

TABLE I. – *The elastic and total cross-sections as computed with complex Yukawa wells.*

$\beta^{-1}\mu$	$\alpha^{-1}\mu$	V_I (MeV)	V_R (MeV)	σ_{tot} (mb)	σ_{el} (mb)
0.83	—	30.2	0	120	30
1	—	23.0	0	120	17.9
1	1	23.0	18.9	139.5	30
1	2	23.0	4.83	137.2	30
2	—	2.58	0	120	3.8
2	1	2.58	27.9	149.5	30
2	2	2.58	6.82	156.1	30

It is seen, even without any real potential, that the imaginary part must have at least a range of $0.83\,\mu^{-1}$. The presence of a real potential still increases the scattering. The maximum depth for a real potential of normal range ($\alpha^{-1}\mu = 1$) is rather small (*). Furthermore, if an attractive real potential is able to increase somewhat the annihilation (total minus elastic) cross-sections, at the same time, the elastic cross-section increases by a comparable amount, and the situation is not improved.

We thus have a strong indication that, in order to fit the experimental results, we need a long range imaginary potential (to provide the large total cross-section) and a small real potential (to obtain a small scattering cross-section).

3. – Semi-theorical complex potential.

The phenomenological real potentials established in the preceding section are much weaker than the Ball and Chew real potential. Thus, it can be anticipated that the latter will give too much scattering. The results with this interaction, however, cannot safely be guessed from Born approximation.

(*) For the sake of comparison, we recall that the depth of the Yukawa well of the same range which fits the deuteron is 47 MeV.

Furthermore, since the Signell and Marshak potential worked so nicely in the proton-proton case, it is probably worthwhile to attempt to consider it in the present antinucleon problem.

Consequently, we first investigate the following model: the real part of the potential, at least for distances larger than $r_c = 0.38\mu^{-1}$, will be the Signell and Marshak potential, with the sign of the 2nd order term changed, as in the model of Chew and Ball. To this real potential, we add the same phenomenological imaginary Yukawa shaped potential as in Sect. **2**. We try to adjust the range β^{-1} and the depth V_I so as to obtain a total cross-section of about 150 mb, and an annihilation cross-section not much smaller, for an incident energy of 167 MeV.

Various models for the core region have been used. Either we describe it by a complex well, the depths of which have been extensively varied in signs and magnitudes, or we simulate the core by absorption boundary conditions at r_c.

In contrast with the nucleon-nucleon system, in the nucleon-antinucleon case we must consider, for given L and S, both isotopic spins 0 and 1. From the corresponding scattering amplitudes M_0 and M_1 one obtains the differential cross-sections for the various processes ([10]):

$$\frac{\mathrm{d}\sigma_{\mathrm{p}+\bar{\mathrm{p}}\to\mathrm{p}+\bar{\mathrm{p}}}}{\mathrm{d}\Omega} = \left|\frac{M_0 + M_1}{2}\right|^2,$$

$$\frac{\mathrm{d}\sigma_{\mathrm{p}+\bar{\mathrm{p}}\to\mathrm{n}+\bar{\mathrm{n}}}}{\mathrm{d}\Omega} = \left|\frac{M_0 - M_1}{2}\right|^2,$$

$$\frac{\mathrm{d}\sigma_{\mathrm{n}+\bar{\mathrm{p}}\to\mathrm{n}+\bar{\mathrm{p}}}}{\mathrm{d}\Omega} = |M_1|^2 .$$

For each partial wave, the computation of M_0 and M_1 is performed with the complex phase-shift method ([11]) for tensor forces, extended to the situation where there is also a complex potential. In the cases of a singlet state and of a triplet $L = J$ state, the real and the imaginary parts of the radial wave functions satisfy two differential equations coupled through the imaginary part of the potential. For the other triplet states, the wave functions are also coupled by the tensor force, and we have a system of four coupled real differential equations. The principal formulae are given in Appendix II.

The calculation of explicit solutions of these differential equations, the determination of proper combinations obeying the various boundary conditions at the core radius, the calculation of the corresponding phase-shifts,

([10]) H. A. Bethe and J. Hamilton: *Nuovo Cimento*, **4**, 1 (1956).

([11]) W. Rarita and J. Schwinger: *Phys. Rev.*, **59**, 436, 556 (1941).

and finally the computation of the cross-sections have been carried out on the IBM 704 digital computer. For $L > 3$, the contributions of the real potential are negligible, and we used the second order Born approximation for the imaginary potential.

The total cross-section for the $p + \bar{p}$ reactions is the sum of three terms:

a) elastic cross-section σ_{el} (final state $p + \bar{p}$);

b) exchange cross-section σ_{ex} (final state $n + \bar{n}$);

c) annihilation cross-section σ_{an}.

The results for various values of the depth and of the range of the imaginary Yukawa potential are tabulated in Table II. The ratio $\varrho = \sigma_{an}/\sigma_{total}$ is also presented. The calculations were performed with 15 different boundary conditions at the core, in order to test the sensitivity of the results to the structure of the core. We only report three sets of results corresponding respectively to the black sphere, to the most favourable case, and to the least favourable one.

The condition I, which is of the same kind as in BALL and CHEW, is such that there are only ingoing waves at the surface of the core, where the logarithmic derivatives of the reduced wave function must be $-ik$ (k is chosen here as the wave number at infinity). For condition II, the core is a complex square well with the parameters $V_R = 650$ MeV, $V_I = 1000$ MeV. Condition III corresponds to a real infinite repulsive core.

The different cross-sections should in principle depend on the initial conditions. But, in the presence of an imaginary potential of sufficient importance, the wave function is damped before it reaches the core. Since the phenomena in this region are little known, it is most satisfactory to obtain results which are fairly independent of the boundary conditions.

The elastic cross-section is practically independent of the presence of an imaginary potential. It essentially comes from the real part of the potential and thus it appears that it is impossible to obtain a small scattering cross-section.

We cannot give serious consideration to the results concerning the exchange cross-section because the latter proceeds from the difference between the amplitudes M_0 and M_1 whereas, in our phenomenological model, the imaginary potential does not depend on the isotopic spin. Nevertheless, the exchange cross-section also seems to be too large, since experimentally it is only of the order of a few millibarns.

The annihilation cross-section is extremely sensitive to the imaginary potential. If we want to obtain a large ratio of annihilation to total cross-section, we must use a strong imaginary potential. As a consequence, the total cross-section becomes much too large. On the other hand, if we adjust the depth of the imaginary potential in order to obtain a reasonable total cross-section,

TABLE II. – *Different cross-sections as computed with the semi-theoretical complex potential.*

Range β^{-1} (μm)	Imaginary depth V_I (MeV)	Initial con-ditions	$\bar{p}$-p cross-sections (mb)					$\bar{p}$-n cross-sections (mb)			
			elastic	ex-change	annihil-ation	total	ϱ=annihil-ation/total	elastic	annihil-ation	total	ϱ=annihil-ation/total
2	10	I	80	14	391	485	.81	64	396	460	.86
		II	75	13	394	482	.82	64	397	461	.86
		III	74	17	386	477	.81	63	397	460	.86
	3	I	83	33	150	266	.56	112	160	272	.59
		II	68	23	160	251	.64	79	171	250	.68
		III	50	34	128	212	.60	49	132	181	.73
	1	I	83	38	70	191	.37	118	80	198	.40
		II	66	27	83	176	.47	80	94	174	.54
		III	48	40	44	132	.33	51	46	97	.47
1	7	I	82	31	73	186	.39	108	83	191	.43
		II	68	22	83	173	.48	77	94	171	.55
		III	50	33	52	135	.39	47	57	104	.55
	0	I	83	42	29	154	.19	123	39	162	.24
		II	66	29	42	137	.31	81	54	135	.40
(*a*) 2	3	II	80	25	158	263	.60	109	171	280	.61
(*b*) 2	1.5	II	50	17	102	169	.60	55	114	169	.67

a) The sign of the spin orbit term is reversed. *b*) The depth of the real potential is multiplied through a factor 0.7.

the maximum value of the ratio $\varrho = \sigma_{an}/\sigma_{tot}$ is 0.5. The experimental ratio is at present much larger.

In addition, we have performed the calculations without any imaginary potential. With condition I, we then have the Ball and Chew model. The exact calculation then gives somewhat less favourable results than those obtained with the W.K.B. approximation. The ratio σ_{an}/σ_{tot} is .19 for $\bar{p}$-p, in strong disagreement with the experimental results. Even with condition II, we cannot obtain a better ratio than .31. The results for the $\bar{p}$-n case are quite similar.

As the spin-orbit term of the potential is a phenomenological one, a run was made with its sign changed. The results are not significantly altered.

Since the real potential seems too strong, a run was made in which it is damped by a factor 0.7. It appears that this reduction is not sufficient; it seems necessary to divide the theoretical real potential which was considered here by at least a factor 2.

4. – Conclusion.

In order to fit the experimental large total cross-sections and small elastic cross-sections, it is necessary to utilize a real potential of moderate strength, but a long range (at least around $\hbar/\mu c$) imaginary potential.

This last requirement could perhaps be met in a model ([8]) inspired by the fixed source meson theory, but improved so as to take also into account annihilation effects. Since the total cross-section is in first approximation proportional to the depth for an imaginary potential (instead of the square of the depth as for a real potential), a relatively small imaginary potential alone would be enough to produce a large total cross-section.

It is much more difficult to obtain a real potential which would be sufficiently small. The straightforward extension of the Gartenhaus potential to the nucleon-antinucleon system predicts a too large real potential. If additional annihilation-type graphs do not provide the destructive interference needed for obtaining a smaller real potential, it will appear difficult to keep the spirit of the fixed source meson theory for the nucleon-antinucleon interaction, even in the relatively low energy domain which was considered here. The necessity of a resort to the relativistic theory seems more and more difficult to discard.

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We gratefully acknowledge Professor M. Lévy for having proposed this problem, and for his continuous interest. We thank Mr. R. A. Bryan for having kindly made available to us tabulations of the Gartenhaus potential.

The numerical computations were performed by courtesy of the IBM-France Company, through their European Research Fund, and we are especially indebted to Miss. F. RATTAUD for her help in programming.

APPENDIX I

The elastic cross-section is obtained from the first-order scattering amplitude $f^{(1)}(\theta)$:

$$\sigma_{el} = \int |f^{(1)}(\theta)|^2 \, d\Omega = 4\pi M^2 \left[\frac{V_R^2}{\alpha^4(\alpha^2 + 4k^2)} + \frac{V_I^2}{\beta^4(\beta^2 + 4k^2)}\right], \tag{1}$$

where k is the initial (and final) momentum magnitude, and $M/2$ the reduced mass. We put $\hbar = c = \mu = 1$.

The total cross-section can be evaluated using the optical theorem. Since the elastic cross-section is of second order in the potential, consistency requires that the forward scattering amplitude be evaluated up to the second Born approximation:

$$\sigma_{tot} = \sigma_{tot}^{(1)} + \sigma_{tot}^{(2)}, \tag{2}$$

where:

$$\sigma_{tot}^{(1)} = 4\pi M V_I/(k\beta^3), \tag{3}$$

$$\sigma_{tot}^{(2)} = 4\pi M^2 \left[\frac{V_R^2}{\alpha^4(\alpha^2 + 4k^2)} - \frac{V_I^2}{\beta^4(\beta^2 + 4k^2)} + \frac{V_R V_I}{k^2 \alpha\beta(\alpha^2 - \beta^2)} \operatorname{tg}^{-1} \frac{2k(\alpha - \beta)}{\alpha\beta + 4k^2}\right]. \tag{4}$$

It is seen by inspection of formulae (1), (3) and (4) that the Born approximation converges (at least up to the second order) when the elastic cross-section is small compared to the total cross-section, which is precisely the situation here.

APPENDIX II

Let us first consider the case of a triplet state $L = J$. The radial wave function has two real components:

$$\varphi_J(kx) = u_J(kx) + i w_J(kx) .$$

The 2 coupled real differential equations obeyed by u_J and w_J admit 2 systems of linearly independent solutions $\varphi^{(1)}$ and $\varphi^{(2)}$, corresponding to different boundary conditions at the core; if this boundary condition is fixed, $\varphi(kx)$ is determined and has the asymptotic form:

$$\varphi(kx) \sim \frac{\exp[i\delta_{JJ}]\sin(kx - J\pi/2 + \delta_{JJ})}{kx},$$

where δ_{JJ} is a complex phase-shift independent of m ($m = \pm 1$). The calculated solution is analysed, outside the range of the potential, in terms of spherical Bessel functions, with two complex constants A and B:

$$\varphi = Aj_J(kx) + Bn_J(kx)\,.$$

By identification we obtain the phase-shift in the useful form $\exp[2i\delta_{JJ}]$:

$$\exp[2i\delta_{JJ}] = \frac{A + iB}{A - iB}\,.$$

For the spherical Bessel functions, the following asymptotic forms are used:

$$j_J(kx) \sim \frac{\sin(kx - J\pi/2)}{kx}\,, \qquad n_J(kx) \sim \frac{\cos(kx - J\pi/2)}{kx}\,.$$

We now study the more complicated case of the triplet states of parity $(-1)^{J+1}$. The waves of orbital angular momenta $L = J \pm 1$ are coupled by the real tensor forces. The radial wave functions:

$$\varphi_J(kx) = u_J^{J+1}(kx) + iw_J^{J+1}(kx)\,,$$
$$\psi_J(kx) = u_J^{J-1}(kx) + iw_J^{J-1}(kx)\,,$$

obey a system of 4 coupled real differential equations, which are easily obtained from those used for a real potential ([12]). For a fixed boundary condition at the core, there are two linearly independent solutions. The general solution is of the form:

$$\varphi = C_1\varphi^{(1)} + C_2\varphi^{(2)}\,,$$
$$\psi = C_1\psi^{(1)} + C_2\psi^{(2)}\,,$$

where C_1 and C_2 are two complex constants.

The asymptotic form of φ and ψ, for a given m, should be

$$\varphi(kx) \sim D^{(m)}_{J\,J+1}\frac{\exp[i\delta^m_{J\,J+1}]\sin(kx - (J+1)\pi/2 + \delta^m_{J\,J+1})}{kx}\,,$$

$$\psi(kx) \sim D^{(m)}_{J\,J-1}\frac{\exp[i\delta^m_{J\,J-1}]\sin(kx - (J-1)\pi/2 + \delta^m_{J\,J-1})}{kx}\,.$$

([12]) M. GOURDIN: *Journ. Phys. Rad.*, **17**, 988 (1956).

The D are constants related to Clebsch-Gordan coefficients:

$$D^0_{J\,J+1} = \quad D^{\pm 1}_{J\,J-1} = \sqrt{\frac{J+1}{2J+1}}\,,$$

$$D^0_{J\,J-1} = -\,D^{\pm 1}_{J\,J+1} = \sqrt{\frac{J}{2J+1}}\,.$$

The last requirement determines the constants C_1 and C_2.

The two independent systems $\varphi^{(1)}$, $\psi^{(1)}$ and $\varphi^{(2)}$, $\psi^{(2)}$ are analysed, as in the previous case, in spherical Bessel functions:

$$\left|\begin{aligned} \varphi^{(1)} &= A^{(1)}_+ j_{J+1}(kx) + B^{(1)}_+ n_{J+1}(kx)\,,\\ \psi^{(1)} &= A^{(1)}_- j_{J-1}(kx) + B^{(1)}_- n_{J-1}(kx)\,,\end{aligned}\right.$$

$$\left|\begin{aligned} \varphi^{(2)} &= A^{(2)}_+ j_{J+1}(kx) + B^{(2)}_+ n_{J+1}(kx)\,,\\ \psi^{(2)} &= A^{(2)}_- j_{J-1}(kx) + B^{(2)}_- n_{J-1}(kx)\,.\end{aligned}\right.$$

We finally obtain 4 phase-shifts: $\delta^0_{J\,J+1}$, $\delta^{\pm 1}_{J\,J+1}$, $\delta^0_{J\,J-1}$, $\delta^{\pm 1}_{J\,J-1}$:

$$\exp\,[2i\delta^0_{J\,J+1}] = \frac{R + iS + 2i\sqrt{J/(J+1)}\,T}{D}\,,$$

$$\exp\,[2i\delta^0_{J\,J-1}] = \frac{R - iS - 2i\sqrt{J/(J+1)}\,U}{D}\,,$$

$$\exp\,[2i\delta^{\pm 1}_{J\,J+1}] = \frac{R + iS - 2i\sqrt{(J+1)/J}\,T}{D}\,,$$

$$\exp\,[2i\delta^{\pm 1}_{J\,J-1}] = \frac{R - iS + 2i\sqrt{J/(J+1)}\,U}{D}\,.$$

R, S, T, U, D are complex numbers, combinations of the A and the B:

$$\begin{aligned} R &= (A^{(1)}_+ A^{(2)}_- - A^{(2)}_+ A^{(1)}_-) + (B^{(1)}_+ B^{(2)}_- - B^{(2)}_+ B^{(1)}_-)\,,\\ S &= (B^{(1)}_+ A^{(2)}_- - B^{(2)}_+ A^{(1)}_-) - (A^{(1)}_+ B^{(2)}_- - A^{(2)}_+ B^{(1)}_-)\,,\\ T &= A^{(1)}_+ B^{(2)}_+ - B^{(1)}_+ A^{(2)}_+\,,\\ U &= A^{(1)}_- B^{(2)}_- - B^{(1)}_- A^{(2)}_-\,,\\ D &= (A^{(1)}_+ - iB^{(1)}_+)(A^{(2)}_- - iB^{(2)}_-) - (A^{(1)}_- - iB^{(1)}_-)(A^{(2)}_+ - iB^{(2)}_+)\,.\end{aligned}$$

There is a relation between T and U, which is of the Wronskian type:

$$T + U = 0\,.$$

It is easy to see that the phase-shifts are not independent and satisfy some relations analogous to those given by Schwinger in the case of a real potential:

$$\exp\,[2i\delta^{\pm 1}_{J\,J+1}] - \exp\,[2i\delta^{0}_{J\,J+1}] = \exp\,[2i\delta^{\pm 1}_{J\,J-1}] - \exp\,[2i\delta^{0}_{J\,J-1}]\,.$$

It is convenient to use a matrix notation for the numerical computation of the wave function.

The cross-sections are obtained from the phase-shifts by the usual formulae.

Note added in proof.

According to very recent and preliminary experimental results (private communication from O. CHAMBERLAIN), there are indications for some scattering at 150 MeV; the elastic cross-section would lie between 40 and 80 mb. As it has been pointed out in the above paper, it is evident that the larger is the elastic cross-section, the less important is the needed imaginary potential. If the elastic cross-section happens to be of an order as large as 80 mb, the model of Ball and Chew would nicely explain the results, as it can be seen from Table II.

RIASSUNTO (*)

Le sezioni efficaci nucleone-antinucleone sono state calcolate a 167 MeV con due modelli a buca di potenziale complessa. Si sono prima studiate buche complesse di Yukawa in approssimazione di Born. Si è successivamente analizzato sull'ordinatore elettronico IBM 704 un potenziale semiteorico: la parte reale è il potenziale di Signel e Marshak adattato al problema nucleone-antinucleone, e la parte immaginaria una buca fenomenologica di Yukawa. Per render conto dei risultati sperimentali (grande sezione efficace totale e piccola sezione efficace di diffusione), risulta necessario che il potenziale reale sia debole e che il potenziale immaginario abbia un range almeno uguale a quello usuale delle forze nucleari. La teoria mesonica della sorgente fissa da un lato conduce a un potenziale reale troppo forte e d'altra parte non predice alcun potenziale immaginario; questa teoria non può render conto dei risultati sperimentali.

(*) *Traduzione a cura della Redazione.*