# Nuclear Matrix Elements for Some 1st-Forbidden Unique β Transitions.

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**Résumé.** — On étudie dans le cadre du modèle des couches les éléments de matrice nucléaires pour les transitions β interdites du premier ordre uniques, autour du nombre de masse $A=40$. L'analyse des résultats expérimentaux permet d'obtenir des éléments de matrice expérimentaux. Puis on calcule ces éléments de matrice, d'abord avec le modèle simple dans lequel les moments angulaires du groupe des protons et du groupe des neutrons sont séparément de bons nombres quantiques; ensuite avec un modèle plus compliqué. Les résultats du deuxième modèle sont en assez bon accord avec l'expérience en ce qui concerne les valeurs relatives des éléments de matrice; un écart systématique est obtenu pour les valeurs absolues. L'étude de transitions γ analogues permet d'attribuer l'écart, au moins pour sa plus grande part, à des effets de structure nucléaire.

## 1. – Introduction.

It has been established by the work of I. TALMI and his collaborators (1-3) that the shell model with *j-j* coupling is quantitatively able to account in a very precise way for the energies of the nuclei, at least in certain regions of the mass number. This for a long time unsuspected agreement could be obtained by avoiding the introduction of poorly known numerical values for the radial wave functions or the two-body forces; the energy matrix elements for

(1) R. THIEBERGER and I. TALMI: *Phys. Rev.*, **102**, 589 (1956).
(2) I. TALMI and R. THIEBERGER: *Phys. Rev.*, **103**, 718 (1956).
(3) S. GOLDSTEIN and I. TALMI: *Phys. Rev.*, **105**, 995 (1957).

simple configurations were instead treated as free parameters, and the energies for more complicated configurations then followed.

Other properties than energies are not expected to be in such as good an agreement with experiment, since the energy is exceptionally favoured by its stationary character. For instance, the nuclear matrix elements for allowed β transitions ([4]) show marked deviations from the $j$-$j$ coupling shell model values. These deviations however are strongly reduced if some configuration mixing is introduced ([5]).

In the present paper, we study the nuclear matrix elements for 1st-forbidden unique β transitions in the mass number region around $A = 40$. The $d_{\frac{3}{2}}$ shell closes at $N$ or $Z = 20$, and is followed by the $f_{\frac{7}{2}}$ shell. These $\beta^-$ radioactive nuclei with a proton number $Z < 20$ and a neutron number $N > 20$ undergo transitions in which a $f_{\frac{7}{2}}$ neutron turns into a $d_{\frac{3}{2}}$ proton. All these transitions (with the exception of the $^{40}$K decay) occur with a spin change of 2 units and with a change of parity: they are unique 1st-forbidden ([6]).

We shall first determine the experimental values of the nuclear matrix elements, through an analysis of the experimental information. We shall then proceed to the calculation of these matrix elements with simple wave functions, and also with more refined wave functions. These results will be discussed. The relation with some γ matrix elements will also be studied.

## 2. – Experimental β nuclear matrix elements.

In the notation of ([6]), the nuclear matrix element for a 1st-forbidden unique transition is:

$$|M|^2 = \sum |\mathcal{Y}_{2m}(\boldsymbol{\sigma})|^2 , \qquad (1)$$

where the summation is on $m$ and on the spin states of the final nucleus. $\mathcal{Y}_{2m}$ is a second-order tensor, bilinear in $\boldsymbol{\sigma}$ and $\boldsymbol{r}$, defined by:

$$\mathcal{Y}_{lm}(\boldsymbol{\sigma}) = \frac{1}{l}\,\boldsymbol{\sigma}\cdot\mathbf{grad}\,\mathcal{Y}_{lm}(\boldsymbol{r}), \qquad (2)$$

where $\mathcal{Y}_{lm}(\boldsymbol{r})$ is the usual solid spherical harmonic.

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([4]) W. C. GRAYSON Jr. and L. W. NORDHEIM: *Phys. Rev.*, **102**, 1093 (1956).

([5]) R. J. BLIN-STOYLE and C. A. CAINE: *Phys. Rev.*, **105**, 1810 (1957).

([6]) For a review of the theory of forbidden β decay, see *e.g.* E. KONOPINSKI, in *Beta- and Gamma-Ray Spectroscopy*, edited by K. SIEGBAHN (Amsterdam, 1955), p. 292. We here follow the notations of this paper.

This matrix element is related to the comparative half-life $f_1 t$ by:

$$f_1 t = \frac{2\pi^3 \ln 2/g^2}{(32\pi/15)\,|M|^2}\,, \tag{3}$$

where $g$ is the Gamow-Teller coupling constant. We adopt the value which results from the most recent measurements of the half-lives of $^{14}O$ (7) and of the neutron (8) which yield $2\pi^3 \ln 2/g^2 = 4\,366$ s, when $M$ in (3) is expressed in units of electron Compton wave length $\hbar/mc$. $f_1$ is defined by DAVIDSON (9),

TABLE I.

| | | log $ft$ | log $f_1t$ | $10^6\,|M|^2_{exp} = 651/f_1t \cdot 10^6$ | Simple wave functions | | Detailed wave functions | |
|---|---|---|---|---|---|---|---|---|
| | | | | | $\varrho$ | $10^6\,|M|^2_{SP.eff}$ | $\varrho$ | $10^6\,|M|^2_{SP.eff}$ |
| ($a$, $b$) | $^{37}S \to {}^{37}Cl$ | 7.23 | 7.93 | 7.7 | 1 | 7.7 | 1 | 7.7 |
| ($c$, 19) | $^{38}Cl \to {}^{38}A$ | 7.42 | 8.19 | 4.20 | 4/5 | 5.2 | 0.80 | 5.2 |
| ($d$) | $^{39}Cl \to {}^{39}A$ | 7.8 | 8.3 | 3.26 | 1/4 | 13.0 | 0.9 | 3.6 |
| ($e$, $f$) | $^{39}A \to {}^{39}K$ | 9.91 | 9.01 | 0.64 | 1/2 | 1.3 | 0.11 | 5.8 |
| ($g$) | $^{41}A \to {}^{41}K$ | 8.34 | 8.50 | 2.15 | 3/8 | 5.7 | 3/8(*) | 5.7 (*) |
| ($h$, 19) | $^{42}K \to {}^{42}Ca$ | 7.95 | 8.42 | 2.48 | 6/5 | 2.1 | 0.66 | 3.8 |
| ($i$) | $^{43}K \to {}^{43}Ca$ | 8.70 | 8.62 | 1.56 | 1 | 1.6 | 0.24 | 6.5 |

$|M|^2$ is in units $\hbar = m = c = 1$

(*) These values come from a very rough estimate of the « detailed wave function ».
($a$) E. BLEULER and W. ZÜNTI: *Helv. Phys. Acta*, **19**, 137 (1946).
($b$) D. STROMINGER, J. M. HOLLANDER and G. T. SEABORG: *Rev. Mod. Phys.*, **30**, 585 (1958).
($c$) J. W. COBBLE and R. W. ATTEBERRY: *Phys. Rev.*, **80**, 917 (1950).
($d$) J. R. PENNING, H. R. MALTRUD, J. C. HOPKINS and F. H. SCHMIDT: *Phys. Rev.*, **104**, 740 (1956).
($e$) A. R. BROSI, H. ZELDES and B. H. KETELLE: *Phys. Rev.*, **79**, 902 (1950).
($f$) H. ZELDES, B. H. KETELLE, A. R. BROSI, C. R. FULTZ and R. F. HIBBS: *Phys. Rev.*, **86**, 811 (1952).
($g$) A. SCHWARTZSCHILD, B. M. RUSTAD and C. S. WU: *Phys. Rev.*, **103**, 1796 (1956).
($h$) P. R. J. BURCH: *Nature*, **172**, 361 (1953).
($i$) T. LINDQVIST and A. C. G. MITCHELL: *Phys. Rev.*, **95**, 444 (1954).

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(7) J. B. GERHART: *Phys. Rev.*, **109**, 897 (1958).

(8) A. N. SOSNOVSKIJ, P. E. SPIVAK, P. E. PROKOFIEV, A. YU, I. E. KUTIKOV and YU. P. DOBRYNIN: *Contribution to the* 1958 *Annual International Conference on High Energy Physics at CERN.*

(9) J. P. DAVIDSON JR.: *Phys. Rev.*, **82**, 48 (1951). This reference is inaccurate in its presentation: if the factor

$$|Q_{n+1}(\boldsymbol{\sigma}, \boldsymbol{r})|^2 = (32\,\pi/15)\,|M|^2$$

is removed from the definition of $C_n$ in eq. (9), the definition (8) of $f_1$ is then the actual one used by DAVIDSON in his computations, and also by us.

who gives an approximate procedure for computing it. We also computed $f_1$ by numerical integration of the theoretical spectrum, and checked that the approximation of DAVIDSON is accurate within a few percent. $f_1$ depends on the $Q$-value for the β transition. Using the experimental values of $Q$ and of the half-life $t$, we list the experimental values of $f_1 t$, for the transitions of interest, in Table I. The value of $ft$, where $f$ is that quantity which is currently used for allowed transitions is also listed for the sake of comparison. We then use (3) to determine an « experimental » value for the squared matrix element, $|M|^2_{\text{exp}}$, which is given in Table I.

## 3. – Calculated β nuclear matrix elements with simple wave functions.

The matrix element for a single particle transition is easily obtained using the definition (2) of $\mathcal{Y}_{2m}(\boldsymbol{\sigma})$. One has for this single particle case ([10]):

$$|M|^2_{\text{S.P.}} = (9/7\pi)\left(\int_0^\infty R_f(r) r R_d(r)\,\mathrm{d}r\right)^2, \tag{4}$$

where $R_f(r)/r$ and $R_d(r)/r$ are the radial wave functions for the $f$ and $d$ states. The integral in (4) cannot be computed without somewhat arbitrary assumptions on these wave functions. However, an estimate of this integral can be easily obtained from some model, for instance oscillator well wave functions. The parameter $\nu$ of the well can be adjusted to fit the Coulomb energy differences between mirror nuclei ([11]) in this region of $A$). In this way we obtain:

$$|M|^2_{\text{S.P.}} = 9/4\pi\nu \approx 37\cdot 10^{-6}(\hbar/mc)^2\,. \tag{5}$$

A numerical integration with wave functions for an infinite square well of radius $R = 1.6\cdot 10^{-13}(40)^{\frac{1}{3}}$ cm ([12]) leads to the value:

$$|M|^2_{\text{S.P.}} = (9/7\pi)\,0.44\,R^2 \approx 36\cdot 10^{-6}(\hbar/mc)^2\,. \tag{5'}$$

We see that the value of $|M|^2_{\text{S.P.}}$ does not depend too much on the model.

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([10]) Cf. R. NATAF: *Journ. Phys. Rad.*, **14**, 72 (1953). There is a misprint in eq. (5) of this reference, which should be read

$$|a_1|^2 = 12\cdot\frac{L(L-1)}{4L^2-1} \qquad \text{if} \quad J > J'.$$

([11]) B. C. CARLSON and I. TALMI: *Phys. Rev.*, **96**, 436 (1954).

([12]) The radius of the infinite well has to be taken somewhat bigger than the radius of the charge distribution. This value of $R$ leads to a mean square radius of $(3/5)^{\frac{1}{2}}\,1.3\cdot A^{\frac{1}{3}}\cdot 10^{-13}$ cm, which is suggested by Coulomb energies in this region (see ref. ([11])).

But the nuclear radius enters by its square, and the uncertainty on this radius would be reflected in $|M|^2_{\text{S.P.}}$.

This single-particle value should be appropriate to the transition ${}^{37}_{16}\text{Cl}_{17} \to {}^{37}_{17}\text{S}_{16}$. For the other cases, we consider the following « simple » wave functions: the protons (neutrons) in the $d_{\frac{3}{2}}$ ($f_{\frac{7}{2}}$) shell are coupled together to a definite angular momentum $J_1$ ($J_2$), equal to 0 if the proton (neutron) is even, to $\frac{3}{2}$ ($\frac{7}{2}$) if the proton (neutron) is odd; $J_1$ and $J_2$ are coupled together to give the total angular momentum $J$. These configurations are given in Table II. The neutrons in the closed $d_{\frac{3}{2}}$ neutron shell play no role in β transitions and can be neglected (13). The transitions we consider are therefore of the type:

$$(j)^m_{J_1'}(j')^n_{J_2'}J' \to (j)^{m+1}_{J_1}(j')^{n-1}_{J_2}J\,.$$

Table II. – *Detailed wave functions.*

| | |
|---|---|
| ${}^{37}_{16}\text{S}_{21}$ | $\lvert f_{\frac{7}{2}}, \frac{7}{2}\rangle$ |
| ${}^{37}_{17}\text{Cl}_{20}$ | $\lvert d_{\frac{3}{2}}, \frac{3}{2}\rangle$ |
| ${}^{38}_{17}\text{Cl}_{21}$ | $\lvert d_{\frac{3}{2}}, f_{\frac{7}{2}}, 2\rangle$ |
| ${}^{39}_{17}\text{Cl}_{22}$ | $0.92\lvert d_{\frac{3}{2}}, (f_{\frac{7}{2}})^2_0, \frac{3}{2}\rangle + 0.39\lvert d_{\frac{3}{2}}, (f_{\frac{7}{2}})^2_2, \frac{3}{2}\rangle$ |
| ${}^{38}_{18}\text{A}_{20}$ | $\lvert (d_{\frac{3}{2}})^2, 0\rangle$ |
| ${}^{39}_{18}\text{A}_{21}$ | $0.95\lvert (d_{\frac{3}{2}})^2_0, f_{\frac{7}{2}}, \frac{7}{2}\rangle + 0.31\lvert (d_{\frac{3}{2}})^2_2, f_{\frac{7}{2}}, \frac{7}{2}\rangle$ |
| ${}^{41}_{18}\text{A}_{23}$ | $\ldots \lvert (d_{\frac{3}{2}})^2_0, (f_{\frac{7}{2}})^3_{\frac{7}{2}}, \frac{7}{2}\rangle + \ldots$ |
| ${}^{39}_{19}\text{K}_{20}$ | $\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}, \frac{3}{2}\rangle$ |
| ${}^{41}_{19}\text{K}_{22}$ | $0.92\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}, (f_{\frac{7}{2}})^2_0, \frac{3}{2}\rangle - 0.38\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}, (f_{\frac{7}{2}})^2_2, \frac{3}{2}\rangle$ |
| ${}^{42}_{19}\text{K}_{23}$ | $0.74\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}, (f_{\frac{7}{2}})^3_{\frac{7}{2}}, 2\rangle + 0.65\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}, (f_{\frac{7}{2}})^3_{\frac{5}{2}}, 2\rangle + 0.17\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}, (f_{\frac{7}{2}})^3_{\frac{3}{2}}, 2\rangle$ |
| ${}^{43}_{19}\text{K}_{24}$ | $0.88\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}, (f_{\frac{7}{2}})^4_0, \frac{3}{2}\rangle - 0.47\lvert (d_{\frac{3}{2}})^3_{\frac{3}{2}}\ (f_{\frac{7}{2}})^4_2, \frac{3}{2}\rangle + \ldots$ |
| ${}^{42}_{20}\text{Ca}_{22}$ | $\lvert (d_{\frac{3}{2}})^4_0, (f_{\frac{7}{2}})^2_0, 0\rangle$ |
| ${}^{43}_{20}\text{Ca}_{23}$ | $\lvert (d_{\frac{3}{2}})^4_0, (f_{\frac{7}{2}})^3_{\frac{7}{2}}, \frac{7}{2}\rangle$ |

The simple wave function is the first term of the detailed wave functions.

The matrix elements for such a transition can be related to the single particle value (4) by the straightforward use of tensor algebra (14) and of coefficients of fractional parentage (15):

$$M/M_{\text{S.P.}} = (-1)^{j+J_2-J_2'}\sqrt{(m+1)n(2J+1)(2j'+1)(2J_1+1)(2J_2'+1)}\cdot \cdot\langle j^{m+1}J_1\{|j^m(J_1')jJ_1\rangle\langle j'^nJ_2'\{|j'^{n-1}(J_2)j'J_2'\rangle\cdot\begin{Bmatrix}J_1 & J_2 & J\\ J_1' & J_2' & J'\\ j & j' & 2\end{Bmatrix}. \tag{6}$$

(13) As a check, we also performed the calculations with the $d_{\frac{3}{2}}$ neutrons taken into account, and obtained the same results.

(14) G. Racah: *Phys. Rev.*, **62**, 438 (1942).

(15) G. Racah: *Phys. Rev.*, **63**, 367 (1943). See eq. (28) of this reference.

The coefficients of fractional parentage for identical particles, $\langle\{|\rangle$, are available ([16,17]). The $9-j$ coefficients $\{\}$ ([18]) can also be computed; incidentally, they reduce to Racah coefficients in the cases where one of the $j$ is zero. The resulting value of the ratio:

$$\varrho = |M|^2/|M|^2_{\text{S.P.}} \tag{7}$$

is listed in Table I.

In order to compare the results to experiment, we define an effective single particle matrix element by the relation:

$$|M|^2_{\text{S.P.eff}} = |M|^2_{\text{exp}}/\varrho \; . \tag{8}$$

This procedure allows us to eliminate the uncertain value (5) or (5′). The theory will be satisfactory if $|M|^2_{\text{S.P.eff}}$ is a constant for the different transitions. The values listed in Table I are actually far from constant. Furthermore, the absolute value of $|M|^2_{\text{S.P.eff}}$ is smaller than the estimate (5) or (5′).

## 4. – Calculated β nuclear matrix elements with more detailed wave functions.

The simple wave functions of the last section gave a poor agreement with experiment, both for the relative and for the absolute values of the matrix elements. It is possible to improve these wave functions by using the freedom on the values of $J_1$ and $J_2$. We now choose wave functions of the form:

$$\sum_{J_1 J_2} \alpha_{J_1 J_2} |(d_{\frac{3}{2}})^m_{J_1} (f_{\frac{7}{2}})^n_{J_2} J\rangle \; , \tag{9}$$

where the summation runs over all possible values of $J_1$ and $J_2$. The coefficients $\alpha_{J_1 J_2}$ should be determined by the diagonalization of an energy matrix.

In the cases where the proton (neutron) number is even, it was argued ([3]) that the excitation of the proton (neutron) configuration requires much energy, and the admixture of such excited states should be negligible. This argument, however, was made for the calculation of binding energies. The calculation of β transitions requires better wave functions, and we now take into account the admixtures of excited states of the even configurations. Of course, admixtures occur more easily for odd configurations, and these will be also considered.

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([16]) A. R. Edmonds and B. H. Flowers: *Proc. Roy. Soc. (London)*, A **214**, 515 (1952).

([17]) W. C. Grayson Jr. and L. W. Nordheim: *Phys. Rev.*, **102**, 1084 (1956).

([18]) G. Racah and U. Fano: *Irreducible Tensorial Sets* (New York, to be published).

The calculation of the energy matrices closely follows in all cases the techniques which were previously used ([3]) in the odd-odd case of $^{42}$K and this calculation will not be described in detail here ([19]). It must however be noted that we do not compute the energy contribution of the forces between identical particles; we rather use the experimental information ([20]) from neighbouring nuclei. For instance, in the case of $^{39}$A, the wave function is of the form:

$$\alpha_0 |(d_{\frac{3}{2}})^2_0 f_{\frac{7}{2}}\, 7/2\rangle + \alpha_2 |(d_{\frac{3}{2}})^2_2 f_{\frac{7}{2}}\, 7/2\rangle \; . \tag{10}$$

The proton-proton energy difference between $(d_{\frac{3}{2}})^2_2$ and $(d_{\frac{3}{2}})^2_0$ is then taken as the experimental excitation energy of the first level $(d_{\frac{3}{2}})^2_2$ of $^{38}$A relative to its ground state $(d_{\frac{3}{2}})^2_0$. The neutron-proton forces are treated as in ([3]) and expressed in terms of the same parameters $V_J$. The binding energies which are computed by this procedure are excellent ([21]). The wave functions so obtained are listed in Table II. The energy matrices of interest are given in the Appendix.

Some remarks are necessary: in the case of $^{43}$K, there are two possible states with $J_2 = 2$ for the neutron configuration $(f_{\frac{7}{2}})^4$. These states have seniorities 2 and 4. The seniority 4 state is expected to have a higher energy but it cannot be identified in an unique way with a given level of $^{44}$Ca; furthermore, there is no element in the energy matrix directly connecting the corresponding unperturbed state of $^{43}$K to the unperturbed ground state of $^{43}$K. The admixture of this state is thus expected to be small. On the other hand, it can be shown that this seniority 4 component would not directly contribute to the β matrix element: this component therefore does not interfere with the other ones, and would only cause a slight second order modification of the coefficients of the other components. For all these reasons, we neglect the possible admixture of the seniority 4 state.

We have omitted the energy matrix and the detailed wave function for $^{41}$A. This is because the energy matrix would be too big, and the various states of the $(f_{\frac{7}{2}})^3$ neutron configuration of $^{41}$A cannot be uniquely identified with the levels of $^{43}$Ca. Some preliminary calculations however show that $^{41}$A is fairly well represented by the simple wave function $|(d_{\frac{3}{2}})^2_0,\ (f_{\frac{7}{2}})^3_{\frac{7}{2}},\ 7/2\rangle$.

It is now possible to compute the β decay matrix elements with the more detailed wave functions (9). Denoting by $M(J_1J_2J'_1J'_2)$ the value computed for $M$ from (6), one immediately sees that:

$$\varrho = |M|^2/|M|^2_{\text{S.P.}} = \Big|\sum_{J_1J_2J'_1J'_2} \alpha_{J'_1J'_2}\alpha_{J_1J_2} M(J_1J_2J'_1J'_2)/M_{\text{S.P.}}\Big|^2 \tag{11}$$

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([19]) B. OQUIDAM: *Thesis* (Doctorat de Spécialité de Physique Théorique - Paris), to be published.

([20]) P. M. ENDT and C. M. BRAAMS: *Rev. Mod. Phys.*, **29**, 683 (1957).

([21]) I. TALMI: private communication.

where $\alpha_{J_1'J_2'}$ and $\alpha_{J_1J_2}$ are the coefficients of the wave function (9) for the initial and final states. The values obtained from (11) are listed in Table I. With the value (11) for $\varrho$, (8) gives new values for $|M|^2_{\text{S.P.eff}}$ which are listed in Table I.

It can be seen that the constancy of $|M|^2_{\text{S.P.eff}}$, is improved by the use of the more detailed wave functions. The progress appears when the last column of Table I is compared with the much more dispersed experimental values $|M|^2_{\text{exp}}$. It must also be remembered that the experimental values are sometimes rather imprecise, and on the other hand that the β matrix elements are sensitive to small changes in the wave function.

## 5. – Discussion.

The approximate consistency of the values for $|M|^2_{\text{S.P.eff}}$ obtained with the detailed wave functions, shows that the shell model, with the refinements of Section **4**, provides a fair account for the relative values of the β matrix elements. Comparison with the results obtained in Section **3** shows that the agreement with experiment is alway improved by the use of the wave functions of Section **4**. It is hoped that the small irregularities which subsist would be removed by further complication of the wave functions, for instance by taking into account the possible admixtures of $f_{\frac{5}{2}}$ orbitals. It must also be remembered that the experimental half-lifes and $Q$ values are often uncertain.

On the other hand, the absolute value of $|M|^2_{\text{S.P.eff}}$ shows a marked departure from the estimates (5) or (5′). It does not seem possible to get entirely rid of this discrepancy by a simple change in the radial wave functions. Everything occurs as if the single particle matrix element was consistently smaller by a factor of about 4 than its estimates (5), (5′). The possible reasons for this apparent « renormalization » could be more easily discussed after a study of some $\gamma$ transitions.

## 6. – Comparison with $\gamma$ transitions.

For all the β transitions which were considered here, the daughter nucleus should have an excited state which is in close correspondence with the initial nucleus in its ground state. The $\gamma$ transition of this excited state to the ground state is closely related to the corresponding β transition, and it is possible to compare their rates in the few cases when the $\gamma$ lifetime has been measured, *i.e.* for the decays of the 1.52 MeV level of $^{39}$A and of the 1.29 MeV of $^{41}$K. The (partial) half-lives are listed in Table III.

TABLE III.

| | $\gamma$ partial half-life | $\lvert M\rvert^2_{\exp}$ from $\gamma$ | Corresponding $\beta$ transition | $\lvert M\rvert^2_{\exp}$ from $\beta$ |
|---|---|---|---|---|
| $^{39}$A (1.52 MeV) | $2.1\cdot10^{-9}$ s | $5.2\cdot10^{-6}$ | $^{39}Cl \to {}^{39}A$ | $3.3\cdot10^{-6}$ |
| $^{41}$K (1.29 MeV) | $6.6\cdot10^{-9}$ s | $3.7\cdot10^{-6}$ | $^{41}A \to {}^{41}K$ | $2.2\cdot10^{-6}$ |

These $\gamma$ transitions with a spin change of 2 and a change of parity should be predominantly of M2 character. We assume that the possible E3 contribution is negligible, as indicated by simple estimates ([22]). The $\gamma$ transition results from the jump of a proton between the $f_{\frac{7}{2}}$ and $d_{\frac{3}{2}}$ orbits. We can take into account some rearrangement of the other particles, using the same model as in Section **3**. The important point however is that within this model, *i.e.* with definite orbitals $f_{\frac{7}{2}}$ and $d_{\frac{3}{2}}$ for the proton states, the M2 $\gamma$ matrix element reduces to the $\beta$ matrix element ([23]). The $\gamma$ transition probability per second is then:

$$T = \frac{4\pi}{75}\,\frac{e^2E^5}{\hbar^4 m_p^2 c^7}\left(\mu_p - \frac{1}{3}\right)^2 \sum |\mathcal{Y}_{2m}(\boldsymbol{\sigma})|^2 \,, \tag{12}$$

where $m_p$ is the proton mass, $\mu_p$ the proton magnetic moment in nuclear magneton units, $E$ the $\gamma$-ray energy. $\mathcal{Y}_{2m}(\boldsymbol{\sigma})$ is here in cm. This formula is used to compute an experimental squared matrix element $|M|^2_{\exp}$, as defined in (1), from the observed $\gamma$-transition probability. The value of $|M|^2_{\exp}$ which is listed in Table III can be compared with the experimental value for the corresponding $\beta$ transition matrix element, which is also repeated in Table III. It is seen that the two $\gamma$ and $\beta$ determinations are not in bad disagreement.

It thus appears that the $\gamma$ matrix element is damped by a factor which is roughly the same as for the $\beta$ matrix element. This fact would tend to prove that most of the « renormalization » of the matrix elements cannot be connected with the field-theoretical nature of the interaction ($\beta$ or $\gamma$) under consideration, since the effect is the same for both interactions, although they are so different in their nature. The « renormalization » should thus mostly be a nuclear structure effect. For instance, if the $d_{\frac{3}{2}}$ and $f_{\frac{7}{2}}$ particles are weakly coupled to collective motions of the core, we expect such « renormalization » effects to occur, although many characters of the spherical shell-model will subsist. This aspect of the question is now being studied in our group.

The difference which subsists between the $\beta$ and the $\gamma$ matrix elements could reasonably result form the nuclear model imperfections. For instance, with an admixture of $f_{\frac{5}{2}}$ orbitals, (12) would no longer be valid.

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([22]) V. F. WEISSKOPF: *Phys. Rev.*, **83**, 1073 (1951).
([23]) S. A. MOSZKOWSKI: p. 390 of ref. ([6]).

To summarize this work, the conclusion of our study is that the relative values of the β decay matrix elements are fairly accounted for by the shell model. The absolute values are consistently too small by a « renormalization » factor of about 4.

Finally, we see that it cannot be shown here that the effective operator which is responsible for 1st-forbidden unique β transitions is significantly different from $\mathcal{Y}_{2m}(\boldsymbol{\sigma})$. We very tentatively suggest that this is perhaps not a trivial statement, since it has been proposed that the axial β interaction could, like the vector one, be of a more complicated nature than the conventional one. It is indicated by our results that the effective β-decay coupling constant that appears for 1st-forbidden unique transitions does not seem to be actually different by more than 50% from the coupling constant for allowed transitions (which was introduced in (3)). It would be interesting to see, by a more careful knowledge of the nuclear structure, if it is possible to establish more precisely the equality of these constants, or if, conversely, they appear as being definitely different.

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## APPENDIX

The energy matrices which do not reduce to a number are the following:

| | $J_2$ | 0 | 2 |
|---|---|---|---|
| $^{39}$Cl | 0 | 2.03 | 0.77 |
| | 2 | 0.77 | 0.56 |

,

| | $J_1$ | 0 | 2 |
|---|---|---|---|
| $^{39}$A | 0 | 2.03 | 0.77 |
| | 2 | 0.77 | − 0.07 |

,

$^{41}$K

| $J_2$ | 0 | 2 |
|---|---|---|
| 0 | 6.08 | — 0.71 |
| 2 | — 0.71 | 4.61 |

,

$^{43}$K

| $J_2$ | 0 | 2 |
|---|---|---|
| 0 | 12.17 | — 0.82 |
| 2 | — 0.82 | 11.06 |

,

The energies which are taken into account here are the neutron proton interaction between $d_{\frac{3}{2}}$ and $f_{\frac{7}{2}}$ shells, and the excitation energy of the even group. Minus the energies are given.

The matrix for $^{42}$K is given in (3). The matrix for $^{41}$A is not given here.

## RIASSUNTO (*)

Si studiano nel quadro del modello a shell gli elementi di matrice nucleare per le transizioni β proibite del primo ordine, uniche, intorno al numero di massa $A = 40$. L'analisi dei risultati sperimentali permette d'ottenere degli elementi di matrice sperimentali. Poi si calcolano questi elementi di matrice, prima col modello semplice in cui i momenti angolari del gruppo dei protoni e del gruppo dei neutroni sono separatamente buoni numeri quantici; poi con un modello più complicato. I risultati del secondo modello sono in sufficiente buon accordo con l'esperienza per quanto riguarda i valori relativi degli elementi di matrice; pei valori assoluti si ottiene uno scarto sistematico. Lo studio delle analoghe transizioni γ permette di attribuire lo scarto, almeno per la maggior parte, ad effetti della struttura nucleare.

(*) *Traduzione a cura della Redazione.*