Progress of Theoretical Physics, Vol. 23, No. 1, January 1960

On the Derivation of the Optical Potential in Infinite Nuclear Matter

B. JANCOVICI*

Institute for Nuclear Study, University of Tokyo

(Received September 19, 1959)

The optical potential, for a nucleon in infinite nuclear matter, is derived, all nucleons and interactions being symmetrically treated. The exchange effects are exhibited; a previously neglected exchange graph, which has an important effect for the real part at low energy, is discussed and numerically computed.

§ 1. Introduction and general formalism

The present note describes a derivation of the optical potential for a nucleon propagating through infinite nuclear matter, all nucleons and interactions being symmetrically treated. Such a formalism exhibits more clearly the exclusion effects, some of which will be expressly studied in the following.

An S-matrix element for elastic scattering can be defined between states $|\boldsymbol{k}\rangle$, consisting of the target in the state $|0\rangle$ of a Fermi gas without interactions ("bare vacuum"), plus an incident particle in a plane wave state of momentum $\boldsymbol{k}$. All interactions V are adiabatically switched on and off, producing both the scattering process and the building up of the real target with interactions ("real vacuum"). The S-matrix element is given by a series expansion

$$\langle \boldsymbol{k}'|S|\boldsymbol{k}\rangle = \left\langle \boldsymbol{k}'\right| V \sum_{n=0}^{\infty} \left(\frac{1}{E-K+i\epsilon} V\right)^{n} \left|\boldsymbol{k}\right\rangle, \tag{1}$$

where K is the kinetic energy operator of the system, and E its initial value. (1) can be represented by a series of graphs,[1] some of which have unlinked parts. It is however well known[2] that

$$\langle \boldsymbol{k}'|S|\boldsymbol{k}\rangle = \langle \boldsymbol{k}'|S|\boldsymbol{k}\rangle_L \langle 0|S|0\rangle, \tag{2}$$

where L means that the sum is to be restricted to linked graphs in expansion (1); we also know that $\langle 0|S|0\rangle$ represents the vacuum to vacuum amplitude (it has a modulus unity), so that $\langle \boldsymbol{k}'|S|\boldsymbol{k}\rangle_L$ is actually the observed particle to particle amplitude when we do not consider the unphysical change of phase of the unperturbed target.

The most general graph in $\langle \boldsymbol{k}'|S|\boldsymbol{k}\rangle_L$ is drawn in Fig. 1, and can be obtained by the iteration

* Pensionnaire de la Maison Franco-Japonaise. Now at Ecole Normale Supérieure, Paris, France.

$$\langle \boldsymbol{k}'|S|\boldsymbol{k}\rangle_L = \left\langle \boldsymbol{k}'\right| \mathcal{V} \sum_{n=0}^{\infty} \left(\frac{1}{E-K+i\epsilon} \mathcal{V} \right)^n \left| \boldsymbol{k} \right\rangle \tag{3}$$

of the irreducible part

$$\langle \boldsymbol{k}'|\mathcal{V}|\boldsymbol{k}\rangle = \left\langle \boldsymbol{k}'\right| V \sum_{n=0}^{\infty} \left(\frac{Q}{E-K+i\epsilon} V \right)^n \left| \boldsymbol{k} \right\rangle_L, \tag{4}$$

where Q is the projector outside the one-particle states. Eq. (4) defines a one-body potential $\mathcal{V}$ which produces the same elastic scattering matrix (3) as the complete interaction V. Therefore $\mathcal{V}$ is the optical potential.

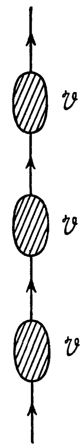

Fig. 1. The general term of $\langle \boldsymbol{k}'|S|\boldsymbol{k}\rangle_L$.

Because of momentum conservation in infinite nuclear matter, (3) is actually of the form $(2\pi)^3\delta(\boldsymbol{k}-\boldsymbol{k}')\mathcal{V}(k)$. k is the wave number of the incident nucleon when it is already inside the nuclear matter, and it is not evident whether k can be unambiguously related to the kinetic energy $\mathfrak{E}$ of the incident nucleon outside the target. Here we just put

$$\mathfrak{E} = k^2/2M + \mathrm{Re}\,\mathcal{V}(k) \tag{5}$$

where M is the nucleon mass.

The first- and second-order graphs of (4) are drawn in Fig. 2. Graphs (1a), (1b), (2a), (2b) depict the interaction of the incident nucleon with one of the target nucleons. These graphs have been previously studied,[3] and their iterations, which are the Brueckner t-matrix, have also been considered.[4] On the other hand, graphs (2c) and (2d) describe modifications in the mutual interaction of two target particles,

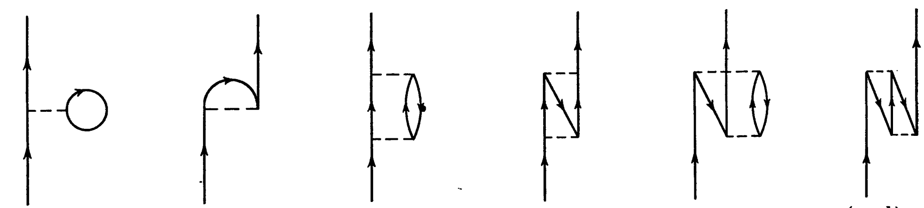

Fig. 2. The first and second order graphs.

for which the incident particle state is no longer available as an intermediate state because of the exclusion principle. These latter graphs did not appear in the treatments[3,4] which neglected the interactions between the target particles. The legitimacy of neglecting these graphs in the Brueckner theory has already been questioned.[5] It may however be noted that these graphs (2c) and (2d) contribute only to the real part of the optical potential, $\mathrm{Re}\,\mathcal{V}$, because the energy denominators cannot vanish: the energy of the excluded intermediate state under consideration is always higher than the initial state energy.

§ 2. Practical computation

We shall now estimate these graphs. The technique of the computation is the same as in Verlet.[3] The Fourier transform of the two-body interaction is $\mathcal{O}J(K)$, where $\mathcal{O}$ is a linear combination of the operators 1, P_σ (spin exchange), P_τ (isotopic spin exchange), $P_\sigma P_\tau$. We now call k_N the momentum of the incident nucleon. The contributions to $\mathcal{V}(k_N)$ of graphs (2c) and (2d) are respectively

$$\mathcal{V}^{(2c)}(k_N)=\frac{\mathrm{Tr}\,\mathcal{O}^2}{2}\frac{M}{(2\pi)^6}\int_> d^3\boldsymbol{k}\int_< d^3\boldsymbol{k}'\int_< d^3\boldsymbol{k}_N'\frac{J^2(|\boldsymbol{k}_N'-\boldsymbol{k}_N|)}{k_N^2+k^2-k_N'^2-k'^2}\delta(\boldsymbol{k}_N'+\boldsymbol{k}'-\boldsymbol{k}_N-\boldsymbol{k}) \tag{6}$$

and

$$\mathcal{V}^{(2d)}(k_N)=\frac{\mathrm{Tr}\,\mathcal{O}^2P_\sigma P_\tau}{2}\frac{M}{(2\pi)^6}\int_> d^3\boldsymbol{k}\int_< d^3\boldsymbol{k}'\int_< d^3\boldsymbol{k}_N'\frac{J(|\boldsymbol{k}_N-\boldsymbol{k}'|)J(|\boldsymbol{k}_N-\boldsymbol{k}_N'|)}{k_N'^2+k'^2-k_N^2-k^2}\times\delta(\boldsymbol{k}_N'+\boldsymbol{k}'-\boldsymbol{k}_N-\boldsymbol{k}), \tag{7}$$

where $>$ and $<$ mean integration on momenta above and below the Fermi momentum k_0.

The exchange operator $\mathcal{O}$ was chosen so that

$$\mathrm{Tr}\,\mathcal{O}^2P_\sigma P_\tau=0, \tag{8}$$

and consequently $\mathcal{V}^{(2b)}$ and $\mathcal{V}^{(2d)}$ vanish. To compute $\mathcal{V}^{(2c)}$, we choose as integration variables $\boldsymbol{K}=\boldsymbol{k}_N'-\boldsymbol{k}_N$, $\boldsymbol{k}$, $\boldsymbol{k}'$; after having performed the integrations on $\boldsymbol{k}'$, $\boldsymbol{k}$, and the angles of $\boldsymbol{K}$, we obtain

$$\mathcal{V}^{(2c)}(k_N)=\mathrm{Tr}\,\mathcal{O}^2(M/2^7\pi^4)\,k_0\int_{k_N-k_0}^{k_N+k_0}dK\,K^2C(K,\,k_N)\,J^2(K), \tag{9}$$

where $C(K, k_N)$ is a function which is given in the Appendix. The integral (9) must be performed numerically.

We used two types of nuclear forces, a gaussian force and a Yukawa force, with the same parameters as in Ref. 3).* The various contributions, including results from Ref. 3), are listed in Table I.

---

* The parameters of the gaussian force as quoted in Ref. 3) (Nuovo Cimento) are inadequate because of some confusion in the units. The force which has actually been used both in Ref. 3) and in the present paper is $\mathcal{O}A\exp(-\alpha^2r^2)$, where $A=72$ Mev and $\alpha=0.674\times10^{13}\mathrm{cm}^{-1}$; the Fermi momentum was $k_0=1.2\times10^{13}\mathrm{cm}^{-1}$. We are indebted to Dr. Verlet for kindly clarifying this point to us.

The exchange dependence, as stated in Ref. 3), involves an important Majorana part.

Table I. The contributions to Re $\mathcal{V}(k)$, in Mev.

		Gaussian interaction			Yukawa interaction		
k/k_0	$\mathfrak{E}$ (Mev)	(1a)+(1b)	(2a)	(2c)	(1a)+(1b)	(2a)	(2c)
1	−21	−44	−15	8	−45	−13	7
1.5	29±1	−28	−12	1.5	−27	−11	0.9
2	90	−20	− 9	0.2	−21	− 8	0.2

§ 3. Discussion

It appears from Table I that, for a low energy incident particle, graph (2c) brings a correction to Re $\mathcal{V}$; although relatively small, this correction is comparable with the other second-order terms and should not be neglected in a consistent second-order calculation.* For an incident particle of higher energy, the exclusion effect (2c) becomes negligible, because, with the regular interactions which were used here the occurrence of self-excitations of the target to high momenta is unlikely. In real nuclear matter, there are however high momenta, due to the actual singular nuclear forces, and it is possible that graph (2c) and higher-order graphs of the same kind would be important at fairly high energies.

In problems in which the first-order graphs vanish, the second-order graph (2c) may happen to be of extreme qualitative importance. Such would be the case in the calculation of the spin-orbit part of the optical potential, from a tensor force interaction.[6,7]

Appendix

$C(K, k_N)$ is defined by

$$2\pi^2 k_0 C(K,\ k_N) = \int_{|\boldsymbol{K}+\boldsymbol{k}_N|<k_0} d\Omega_{\boldsymbol{K}} \int_{\substack{|\boldsymbol{k}-\boldsymbol{K}|<k_0 \\ k>k_0}} \frac{d^3\boldsymbol{k}}{(\boldsymbol{k}-\boldsymbol{k}_N)\cdot\boldsymbol{K}-K^2}. \qquad (\text{A}\cdot 1)$$

— If $k_N - k_0 < K < 2k_0$,

$$\begin{aligned} C(K,\ k_N) = {} & (1/12\, k_0 k_N K^2)[K^4 - 4(k_0+k_N)K^3 + (4k_N + 8k_0 k_N)K^2 \\ & + 4k_0(k_0^2 - k_N^2)K - (k_N^2 - k_0^2)^2] \\ & + (1/3\, k_0 k_N K)[-K^3 + 3\, k_N K^2 + 3(k_0^2 - k_N^2)K + k_N^3 - 3\, k_0^2 k_N - 2\, k_0^3] \\ & \text{Log}|(k_N + k_0)K - K^2| \\ & + (1/24\, k_0 k_N K^4)[K^6 - (3k_N^2 + 9k_0^2)K^4 + 16k_0^3 K^3 + (3k_N^4 + 6k_0^2 k_N^2 \\ & - 9k_0^4)K^2 - (k_N^2 - k_0^2)^3]\text{Log}|(-K^2 + 2k_0 K + k_N^2 - k_0^2)/2| \end{aligned}$$

* A former estimate of graph (2c) in Ref. 5) gave 12 Mev, for a charge and spin independent Yukawa force. The result is very sensitive to the exchange dependence of the interaction.

$$+(1/4\,k_0k_N)(K^2-4k_NK+4k_N^2)\mathrm{Log}|(K^2/2)-k_NK|$$
$$-(1/4\,k_0k_NK^2)(k_N^2-k_0^2)^2\mathrm{Log}|(k_N^2-k_0^2)/2|$$
$$+(1/3\,k_0k_NK)(-k_N^3+3k_0^2k_N+2k_0^3)\mathrm{Log}|(k_N+k_0)K|$$
$$+(1/24\,k_0k_NK^4)[K^6+(3k_N^2-15k_0^2)K^4-16k_0^3K^3$$
$$+(k_N^2-k_0^2)(3k_N^2-15k_0^2)K^2+(k_N^2-k_0^2)^3]\mathrm{Log}|(K^2+2k_0K+k_N^2-k_0^2)/2|. \tag{A·2}$$

— If $2k_0<K<k_N+k_0$,

$$C(K,\ k_N)=(1/6\,k_NK^3)[-K^4+2(k_N^2+k_0^2)K^2-(k_N^2-k_0^2)^2]$$
$$+(1/3\,k_0K)(k_N^2-3k_0^2)\mathrm{Log}|(k_N-k_0)/(k_N+k_0)|$$
$$+(2k_0^2/3k_NK)\mathrm{Log}|(k_N^2-k_0^2)K^2/[(K^2-2k_0K+k_N^2-k_0^2)(K^2+2k_0K+k_N^2-k_0^2)]|$$
$$+(1/24\,k_0k_NK^4)[-K^6+(15k_0^2-3k_N^2)K^4+(-3k_N^4+18k_0^2k_N^2-15k_0^4)K^2$$
$$-(k_N^2-k_0^2)^3]\mathrm{Log}|(K^2-2k_0K+k_N^2-k_0^2)/(K^2+2k_0K+k_N^2-k_0^2)|. \tag{A·3}$$

References

1) J. Goldstone, Proc. Roy. Soc. **239** (1957), 267.
2) See e.g. S. S. Schweber, H. A. Bethe and F. de Hoffmann, Mesons and Fields, vol. I, p. 231.
3) L. Verlet, Nuovo Cimento **7** (1958), 821; Thesis, Ann. de Phys. **4** (1959), 643.
4) L. Verlet and J. Gavoret, Nuovo Cimento **10** (1958), 505.
5) N. M. Hugenholtz and L. Van Hove, Physica **24** (1958), 363.
6) S. Takagi, W. Watari and M. Yasuno, Prog. Theor. Phys. **22** (1959), 549.
7) B. Jancovici, Prog. Theor. Phys. **22** (1959), 585.