PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Radiative Correction to the Ground-State Energy of an Electron in an Intense Magnetic Field

B. JANCOVICI
*Laboratoire de Physique Théorique et Hautes Energies, 91-Orsay, France**

(Received 26 May 1969)

It is pointed out that the ground-state energy of an electron does not deviate much from mc^2, even in the presence of intense magnetic fields larger by several orders of magnitude than 10^{13} G.

FOR astrophysical purposes, it has recently been investigated[1,2] how the ground-state energy E_0 of an electron may be shifted from mc^2 in the presence of a very intense static uniform magnetic field $\mathbf{H}$. Without radiative corrections, E_0 does not depend on H. Assuming that the radiative corrections can be described by adding a term $\mu\boldsymbol{\sigma}\cdot\mathbf{H}$ to the Dirac Hamiltonian [where $\mu=(\alpha/2\pi)(e\hbar/2mc)$ is the Schwinger value for the anomalous magnetic moment, and where the components of $\boldsymbol{\sigma}$ are the Pauli matrices], one finds for the ground-state energy of an electron[3,4]

$$E_0=mc^2|1-(\alpha/4\pi)(e\hbar H/m^2c^3)|\,. \qquad (1)$$

On the basis that (1) vanishes for $H=7.6\times10^{16}$ G and becomes larger than the rest energy of a muon for $H\sim10^{19}$ G, puzzling conclusions about pair creation[1,2] and electron-to-muon decay[2] have been claimed.

We wish to point out that the extrapolation of (1) to high-field values is unjustified. A high field will distort the structure of the electron. The very concept of an anomalous magnetic moment only means that when the energy is expanded with respect to H, the linear term is $\mu\boldsymbol{\sigma}\cdot\mathbf{H}$, but higher-order terms in H become important, of course, if H is large. When H is large, it must be taken into account to all orders; the expansion with respect to the small fine-structure constant $\alpha\approx1/137$ is, however, still legitimate. Therefore, one must compute the Feynman graph of Fig. 1, where the double line represents the propagation of an electron in the external field H.

This computation has actually been carried out many years ago. An essential ingredient is the electron propagator in a uniform magnetic field, which has been independently derived by several authors[5–8] in different but equivalent forms. The Feynman graph itself can then be computed. Most authors at some stage went to the low-field limit, which was the only one in which they were interested. A result which is valid for all field strengths has, however, been given[9] as the following integral representation:

$$E_0=mc^2+\frac{\alpha}{4\pi}mc^2\int_0^1 dv\int\frac{dw}{w}\frac{w}{|w|}\times e^{ivw}\left(\frac{2iLw(ve^{2iLw}+1)}{ve^{2iLw}+2iLw(1-v)-v}-(1+v)\right), \qquad (2)$$

where $L=e\hbar H/m^2c^3$.

When L is small, (2) can be expanded in L and yields (1).[10] When L is large, however, one must look for the asymptotic behavior of (2). Using $z=-iLvw$ as a new variable, one finds for the integral in (2)

$$I=2\int_0^\infty dz\,e^{-z/L}\int_0^1 dv\times\left(\frac{2(1+ve^{-2z/v})}{2z(1-v)+v^2(1-e^{-2z/v})}-\frac{1+v}{z}\right). \qquad (3)$$

When L is large, the leading term in this Laplace transform is obtained by replacing the integrand by its asymptotic form for large z:

$$I\sim2\int_1^\infty dz\,e^{-z/L}\int_0^1\frac{2dv}{2z-2zv+v^2}$$
$$=2\int_1^\infty dz\,e^{-z/L}\frac{2}{[z(z-2)]^{1/2}}\tanh^{-1}\left[\left(\frac{z-2}{z}\right)^{1/2}\right]$$
$$\sim2\int_1^\infty dz\,e^{-z/L}\frac{\ln z}{z+1}=e^{1/L}[\mathrm{Ei}(-1/L)]^2\sim(\ln L)^2 \qquad (4)$$

FIG. 1. Feynman graph for the radiative correction to the energy. The double line represents the propagation of an electron in the external field H.

* Laboratoire associé au Centre National de la Recherche Scientifique.
[1] R. F. O'Connell, Phys. Rev. Letters **21**, 397 (1968); Phys. Letters **27A**, 391 (1968).
[2] H. Y. Chiu and V. Canuto, Astrophys. J. **153**, L157 (1968); V. Canuto and H. Y. Chiu, Phys. Rev. **173**, 1220 (1968); H. Y. Chiu, V. Canuto, and L. Fassio-Canuto, *ibid.* **176**, 1438 (1968).
[3] M. H. Johnson and B. A. Lippmann, Phys. Rev. **77**, 702 (1949).
[4] I. M. Ternov, V. G. Bagrov, and V. Ch. Zhukovskii, Moscow Univ. Bull. **21**, 21 (1966).
[5] J. Schwinger, Phys. Rev. **82**, 664 (1951).
[6] G. Geheniau and M. Demeur, Physica **17**, 71 (1951).
[7] R. Kaitna and P. Urban, Nucl. Phys. **56**, 518 (1964).
[8] G. Geheniau and F. Villars, Helv. Phys. Acta **23**, 178 (1950).
[9] M. Demeur, Acad. Roy. Belg., Classe Sci., Mem. **28**, No. 1643 (1953).
[10] Some higher-order terms in the L expansion of Ref. 9 have mistakes, which have been corrected by R. G. Newton, Phys. Rev. **96**, 523 (1954).

(the last integral has been taken from a table[11]). Therefore, E_0 behaves like $(\ln H)^2$ for large values of H. A more careful, tedious, but straightforward study of (3), with the use of majorizations and minorizations, gives the following more precise result for the asymptotic behavior of E_0:

$$E_0=mc^2+(\alpha/4\pi)mc^2\{[\ln(2e\hbar H/m^2c^3)-C-\tfrac{3}{2}]^2+A+\cdots\},\quad(5)$$

where $C=0.577$ is Euler's constant, and where A is a numerical constant for which we have only found bounds: $-6<A<7$.

One readily sees from (3) that even for tremendous values of H (the characteristic field $m^2c^3/e\hbar$ being 4.4×10^{13} G), the radiative correction to E_0 remains of relative order α. In particular, E_0 certainly does not vanish at $H=(4\pi/\alpha)(m^2c^3/e\hbar)=7.6\times10^{16}$ G, a field value for which (1) is not valid. Some doubts about the limits of validity of the anomalous magnetic moment concept have actually been raised by the authors of Ref. 2 themselves.

[11] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products*, edited by A. Jeffrey (Academic Press Inc., New York, 1965).

PHYSICAL REVIEW VOLUME 187, NUMBER 5 25 NOVEMBER 1969

Interpretation of a Unified Theory of Gravitation and Symmetry Breaking*

DAVID PEAK AND AKIRA INOMATA
Department of Physics, State University of New York, Albany, New York 12203
(Received 15 July 1969)

The formalism of Moen and Moffat is interpreted as a Yang-Mills theory set in a space-time generally endowed with curvature and torsion.

IN a recent paper,[1] Moen and Moffat describe the possibility of a generalized definition of "parallel" transport of a vector nonet [an element of the tensor representation of the combined group of space-time and $U(3)$ transformations] resulting in (a) a connection between space-time and internal symmetries without reference to a "supergroup" and (b) unitary symmetry breaking induced by the presence of a zero-mass boson (to first approximation). We show that it is possible to interpret the formalism in this work as an extended Yang-Mills theory. From this point of view we see that a total symmetry group is already "embedded" in the theory, and that the character of the background space-time is sufficient to break the internal symmetry.

To see how it may be possible to make the aforementioned interpretation, we first review some aspects of a local gauge theory set in a curved background. At the outset there is, presumably, a matter field which displays a unitary symmetry characterized by[2]

$$\psi'(x)=S^{-1}(x)\psi(x).\quad(1)$$

The entities generically designated S are taken to be matrix representations of elements of a group of internal transformations, and are by assumption functions of the space-time coordinates of the event point at which the transformation is made. The internal degrees of freedom of the ψ field are thus adjustable at all other points of space-time, in keeping with the requirements of a local picture of interaction. To ensure the invariance of the dynamical structure of this system, it is necessary to introduce auxiliary field operators B_μ that couple universally with the various ψ components, and which transform under local internal group action as

$$B'_\mu=S^{-1}(B_\mu S-\nabla_\mu S).\quad(2)$$

Here ∇_μ denotes the relevant space-time covariant derivative with respect to the μth coordinate.

In a sense, the B_μ fields are like components of an affine connection[3]; as a consequence, we may define a totally covariant derivative operator expressed symbolically as

$$D_\mu=\nabla_\mu+B_\mu.\quad(3)$$

D_μ commutes with both space-time and internal transformations, and serves to establish a meaning for a parallel transport of fields with mixed indices. In terms of the vector nonets mentioned in I, the operation of D_μ provides, for example,

$$D_\nu A^{\sigma i}=\nabla_\nu A^{\sigma i}+B_\nu{}^i{}_j A^{\sigma j}=\partial_\nu A^{\sigma i}+\left\{\begin{matrix}\sigma\\ \mu\nu\end{matrix}\right\}A^{\mu i}+B_\nu{}^i{}_j A^{\sigma j},\quad(4)$$

where Greek indices refer to space-time structure, Latin indices to internal.

Now, the covariant derivative defined in I is just such an operator, that is, it measures the effect of the total variation of fields. As expressed in that work, the

* Supported in part by the National Science Foundation.
[1] I. O. Moen and J. W. Moffat, Phys. Rev. **179**, 1233 (1969); herein this paper shall be referred to as I.
[2] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).
[3] See, e.g., J. L. Anderson, *Principles of Relativity Physics* (Academic Press Inc., New York, 1967), p. 44.