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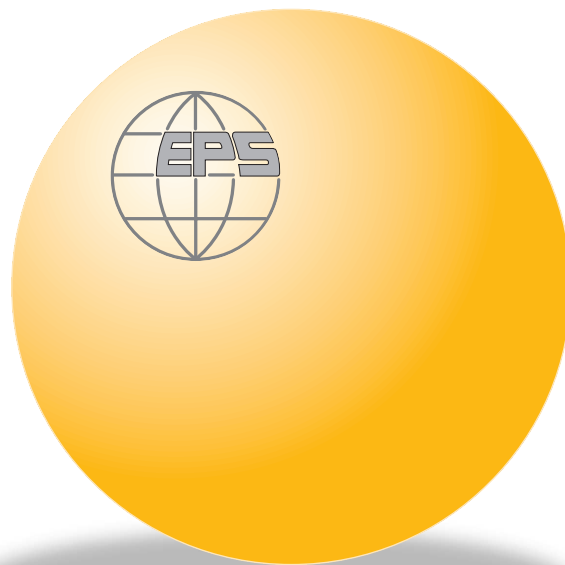
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B. JANCOVICI and L. ŠAMAJ



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Casimir force between two ideal-conductor walls revisited

B. JANCOVICI^{1(*)} and L. ŠAMAJ^{1,2(**)}

¹ *Laboratoire de Physique Théorique (Unité Mixte de Recherche no. 8627, CNRS)
Université de Paris-Sud - Bâtiment 210, 91405 Orsay Cedex, France*

² *Institute of Physics, Slovak Academy of Sciences
Dúbravská cesta 9, 845 11 Bratislava, Slovakia*

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Abstract. – The high-temperature aspects of the Casimir force between two neutral conducting walls are studied. The mathematical model of “inert” ideal-conductor walls, considered in the original formulations of the Casimir effect, is based on the universal properties of the electromagnetic radiation in the vacuum between the conductors, with zero boundary conditions for the tangential components of the electric field on the walls. This formulation seems to be in agreement with experiments on metallic conductors at room temperature. At high temperatures or large distances, at least, fluctuations of the electric field are present in the bulk and at the surface of a particle system forming the walls, even in the high-density limit: “living” ideal conductors. This makes the enforcement of the inert boundary conditions inadequate. Within a hierarchy of length scales, the high-temperature Casimir force is shown to be entirely determined by the thermal fluctuations in the conducting walls, modelled microscopically by classical Coulomb fluids in the Debye-Hückel regime. The semi-classical regime, in the framework of quantum electrodynamics is studied in the companion letter by BUENZLI P. R. and MARTIN Ph. A., *The Casimir force at high temperature (Europhys. Lett., 72 (2005))*.

This letter is related to the one by Buenzli and Martin [1]. For the sake of completeness, we cannot avoid repeating a few things.

Casimir showed in his famous paper [2] that fluctuations of the electromagnetic field in vacuum can be detected and quantitatively estimated via the measurement of a macroscopic attractive force between two parallel *neutral* metallic plates; for a nice introduction to the Casimir effect see [3] and for an exhaustive review see [4].

Let us recall briefly, within the formalism of ref. [3], some aspects of the usual theory for plates considered as made of ideal conductors, which are relevant in view of the present letter. We consider the 3D Cartesian space of points $\mathbf{r} = (x, y, z)$, where a vacuum is localized in the subspace $\Lambda = \{\mathbf{r} | x \in (-d/2, d/2); (y, z) \in R^2\}$ between two ideal-conductor walls (thick slabs) at a distance d from each other. The time-dependent electric $\mathbf{E}(\mathbf{r}, t)$ and magnetic $\mathbf{B}(\mathbf{r}, t)$

(*) E-mail: Bernard.Jancovici@th.u-psud.fr

(**) E-mail: fyzimaes@savba.sk

fields in Λ are the solutions of the Maxwell equations in vacuum, subject to the boundary conditions that the tangential components of the electric field vanish at the ideal-conductor walls $\partial\Lambda = \{\mathbf{r}|x = \pm d/2; (y, z) \in R^2\}$:

$$E_y(\mathbf{r}, t) = E_z(\mathbf{r}, t) = 0 \quad \text{for } \mathbf{r} \in \partial\Lambda. \quad (1)$$

Note that this mathematical definition of the ideal-conductor wall is based on macroscopic electrostatics: the electric field is considered to be zero, without any fluctuation, inside the walls which have no microscopic structure and act only as fixing the instantaneous boundary conditions of type (1). We shall call such a mathematical model of an ideal conductor ‘‘inert ideal conductor’’. For each separate mode labeled by the wave number $\mathbf{k} = (k_x, k_y, k_z)$ with $k_x = \pi n_x/d$ ($n_x = 0, 1, 2, \dots$) and polarization indices $\lambda = 1, 2$ (only one polarization is possible when $k_x = 0$), the quantized energy spectrum of the electromagnetic field between the walls corresponds to that of an oscillator with the frequency $\omega_{\mathbf{k}} = c|\mathbf{k}|$ (c is the velocity of light). At zero temperature $T = 0$, no photons are present and so each mode contributes by the zero-point energy $\hbar\omega_{\mathbf{k}}/2$, where \hbar is Planck’s constant. The d -dependent part of the system ground-state energy leads to the following attractive Casimir force per unit surface of one of the walls:

$$f^0(d) = -\frac{\pi^2\hbar c}{240d^4}. \quad (2)$$

At nonzero temperature $T > 0$, all numbers of photons are possible and each mode contributes by the free energy of the thermalized harmonic oscillator. The Casimir force then reads

$$f^T(d) = -\frac{2}{\pi\beta} \sum'_{n=0} \int_0^\infty dk_\perp k_\perp q_n (e^{2dq_n} - 1)^{-1}, \quad (3)$$

where $\beta = 1/(k_B T)$ is the inverse temperature, the prime in the sum over $n = 0, 1, 2, \dots$ means that the $n = 0$ term should be multiplied by $1/2$, k_\perp is the magnitude of a wave vector component in the (y, z) -plane and $q_n^2 = k_\perp^2 + \xi_n^2/c^2$ with $\xi_n = 2\pi n/(\hbar\beta)$ being the Matsubara frequencies. By a simple change of variables, formula (3) can be rewritten as follows:

$$f^T(d) = -\frac{1}{4\pi\beta d^3} \sum'_{n=0} \int_{nt}^\infty dy y^2 \frac{1}{e^y - 1}, \quad (4)$$

where

$$t = \frac{4\pi d}{\hbar c\beta} \quad (5)$$

is the dimensionless parameter which measures the ratio of the separation between the conductor walls to the thermal wavelength of a photon. The small values of t correspond to low temperatures or small distances where quantum effects dominate. Using the Euler-MacLaurin sum formula, one obtains from eq. (4) the small- t expansion of the form

$$f^T(d) = -\frac{\pi^2\hbar c}{240d^4} - \frac{\pi^2}{45(\hbar c)^3\beta^4} + \frac{1}{\beta d^3} O(e^{-4\pi^2/t}), \quad t \rightarrow 0. \quad (6)$$

It is interesting that the leading correction to the $T = 0$ result (2) is negligible in the experiments which have been performed at room temperature, see for example refs. [5, 6]. The experiments are in good agreement with (6). The large values of t correspond to high temperatures or large distances where the classical limit of quantum mechanics provides an adequate

system description. In the large- t limit, the $n = 0$ term dominates in the sum (4), which implies the classical \hbar -independent leading behavior

$$f^T(d) = -\frac{\zeta(3)}{4\pi\beta d^3} + \frac{1}{\beta d^3} O(e^{-t}), \quad t \rightarrow \infty. \quad (7)$$

For the present time, the high- t region is not accessible to experiments on metals. However, the high-temperature regime might be of interest for electrolytes.

Lifshitz [7] considered the more general case of dielectric walls with a frequency-dependent dielectric permittivity $\epsilon(\omega)$. His starting point was the fluctuations within the walls, which therefore were not considered as inert. He derived the following formula for the Casimir force [4]:

$$f^T(d) = -\frac{1}{\pi\beta} \sum_{n=0}^{\infty} ' \int_0^{\infty} dk_{\perp} k_{\perp} q_n \times \\ \times \left\{ [r_{\parallel}^{-2}(\xi_n, k_{\perp}) e^{2dq_n} - 1]^{-1} + [r_{\perp}^{-2}(\xi_n, k_{\perp}) e^{2dq_n} - 1]^{-1} \right\}, \quad (8)$$

where r_{\parallel} and r_{\perp} are the reflection coefficients of the TM and TE modes, respectively. They are given by

$$r_{\parallel}^{-2}(\xi_n, k_{\perp}) = \left[\frac{\epsilon(i\xi_n)q_n + k_n}{\epsilon(i\xi_n)q_n - k_n} \right]^2, \quad r_{\perp}^{-2}(\xi_n, k_{\perp}) = \left(\frac{q_n + k_n}{q_n - k_n} \right)^2, \quad (9)$$

with $k_n^2 = k_{\perp}^2 + \epsilon(i\xi_n)\xi_n^2/c^2$. When $\epsilon(\omega) < \infty$, eqs. (8) and (9) are well defined. When $\epsilon(\omega) \rightarrow \infty$, the zero-frequency $n = 0$ term in the sum on the r.h.s. of (8) is not uniquely defined because its value depends on the order of the limits $\epsilon(\omega) \rightarrow \infty$ and $n \rightarrow 0$. In order to restore the inert ideal-conductor result (3) based on the electrostatic boundary conditions (1), Schwinger *et al.* [8] postulated the following order: set first $\epsilon(\omega) = \infty$, then take the limit $n = 0$. This prescription implies the reflection coefficients of the zero mode to be $r_{\parallel}^2(0, k_{\perp}) = r_{\perp}^2(0, k_{\perp}) = 1$ for inert ideal metals.

Experiments are performed on real conductors composed of quantum particles, with finite static conductivity σ and plasma frequency ω_p , given by $\omega_p^2 = 4\pi e^2 n/m$, where n is the number density of free electrons of mass m . For such real conductors, one has the Drude formulae for the frequency-dependent $\epsilon(\omega)$:

$$\epsilon(\omega) \sim \frac{4\pi i\sigma}{\omega} \quad \text{for } \omega \rightarrow 0, \quad (10)$$

$$\epsilon(\omega) \sim 1 - \frac{\omega_p^2}{\omega^2} \quad \text{for } \omega \gg \omega_p^2/(4\pi\sigma). \quad (11)$$

The consideration of a frequency-dependent $\epsilon(\omega)$ enables one to avoid an artificial prescription for the order of limits: it is the dynamics of the particle system which ‘‘chooses’’ the correct treatment of the zero-mode contribution. In a series of recent works [9–13], the Drude formula (10) was substituted into eq. (9) considered for the zero Matsubara frequency $\xi_0 \rightarrow 0$. This leads to the reflection coefficients $r_{\parallel}^2(0, k_{\perp}) = 1, r_{\perp}^2(0, k_{\perp}) = 0$ independent of σ , *i.e.* for $n = 0$ the second term on the r.h.s. of eq. (8) does not contribute to the Casimir force for a real conductor. As a mathematical consequence, the additional term $\zeta(3)/(8\pi\beta d^3)$ appears in the Casimir force in any regime. In particular, the large-temperature formula (7) is modified to

$$f_L^T(d) \sim -\frac{\zeta(3)}{8\pi\beta d^3} \quad \text{for } t \rightarrow \infty, \quad (12)$$

a result identical to the one given by Lifshitz [7]. Although the additional term vanishes at zero temperature, it is relevant in the region of small temperatures where it is the source of some contradictions. Namely, it was argued in another series of works [14–16] that, at low temperatures, the relation (11) should be used. An intensive polemic about the low-temperature Casimir effect persists in our days [13,16].

In this letter, we shall concentrate on the high-temperature aspects of the Casimir effect. There is an apparent discrepancy by a factor 1/2 between the high-temperature Schwinger formula (7), valid for inert ideal-conductor walls with the boundary conditions (1), and the Lifshitz formula (12), valid for real-conductor walls with $\epsilon(\omega)$ given by the Drude dispersion relation (10). We aim at explaining this discrepancy on the basis of some exact results for specific microscopic particle systems which are used to model the conductor walls. The consideration of the Casimir effect in the $t \rightarrow \infty$ limit is also motivated by two fundamental simplifications of these model systems. First, according to the correspondence principle, in a microscopic model of matter coupled to electromagnetic radiation at equilibrium, both matter and radiation can be treated classically in the high-temperature limit. This fact manifests itself as the absence of \hbar in the leading terms of the expansions (7) and (12). Second, the application of the Bohr-van Leeuwen theorem [17,18] leads to the decoupling between classical matter and radiation, and to an effective elimination of the magnetic forces in the matter (for a nice detailed treatment of this subject, see ref. [19]). The absence of relativistic effects is seen via the independence of the leading terms in eqs. (7) and (12) from c . We conclude that the matter can be treated in the $t \rightarrow \infty$ limit as a classical matter, unaffected by radiation, where the charges interact only via the instantaneous Coulomb potential.

As a model system of the classical Coulomb fluid, we consider a general mixture of M species of mobile pointlike (structureless) particles $\alpha = 1, 2, \dots$ with the corresponding masses m_α and charges $Z_\alpha e$, where e is the elementary charge and Z denotes integer valence ($Z = -1$ for an electron). Its statistical mechanics is treated in the grand-canonical ensemble characterized by the inverse temperature β and by the species fugacities $\{z_\alpha\}$ or, equivalently, the bulk species densities $\{n_\alpha\}$ constrained by the neutrality condition $\sum_\alpha Z_\alpha n_\alpha = 0$. The thermal average will be denoted by $\langle \dots \rangle$. We use Gaussian units. The interaction energy of particles $\{i\}$ with charges $\{q_i\}$, localized at spatial positions $\{\mathbf{r}_i\}$, is given by $\sum_{i < j} [q_i q_j v(|\mathbf{r}_i - \mathbf{r}_j|) + u_{(\lambda_i + \lambda_j)/2}(|\mathbf{r}_i - \mathbf{r}_j|)]$, where $v(r) = 1/r$ is the Coulomb potential and

$$u_\lambda(r) = \begin{cases} \infty & \text{for } r < \lambda, \\ 0 & \text{for } r \geq \lambda, \end{cases} \quad (13)$$

is the hard-core repulsion potential which prevents the classical thermodynamic collapse between oppositely charged particles. To make the correspondence with the quantum-mechanical version of the model, the hard-core diameter of particles of type α has to be set equal to the thermal de Broglie wavelength $\lambda_\alpha = \hbar(2\pi\beta/m_\alpha)^{1/2}$ [20].

We would like to emphasize that the present particle system represents a microscopic model of “living conductors” where the charge density and the corresponding electric potential/field fluctuate, even for extreme values of physical parameters like the particle density. To be more precise, let us consider the truncated charge-charge correlation function

$$S(\mathbf{r}, \mathbf{r}') = \langle \hat{\rho}(\mathbf{r}) \hat{\rho}(\mathbf{r}') \rangle^T, \quad (14)$$

where the microscopic charge density $\hat{\rho}$ is defined by $\hat{\rho}(\mathbf{r}) = \sum_\alpha Z_\alpha e \hat{n}_\alpha(\mathbf{r})$ with $\hat{n}_\alpha(\mathbf{r}) = \sum_i \delta(Z_\alpha e, q_i) \delta(\mathbf{r} - \mathbf{r}_i)$ being the microscopic number density of α -species. In the infinite space (bulk regime), the fact that the Fourier transform of the Coulomb interaction has the

form $\tilde{v}(\mathbf{k}) = 4\pi/k^2$ implies the following small- k behavior of the charge structure function (the Fourier transform of (14) with respect to $|\mathbf{r} - \mathbf{r}'|$):

$$\tilde{S}(k) = \frac{1}{4\pi\beta}k^2 + O(k^4); \quad (15)$$

for a review of sum rules for charged systems, see ref. [21]. This exact result is not influenced by short-range interaction potentials like the hard-core one. Thus, the second moment of $S(r)$ *does not* depend on the total particle number density $n = \sum_{\alpha} \langle \hat{n}_{\alpha}(\mathbf{r}) \rangle$ and survives also in the high-density region. An immediate consequence of eq. (15) is an asymptotic formula for the long-ranged potential-potential correlation function [22]:

$$\beta \langle \hat{\phi}(\mathbf{r}) \hat{\phi}(\mathbf{r}') \rangle^T \sim \frac{1}{|\mathbf{r} - \mathbf{r}'|} \quad \text{as } |\mathbf{r} - \mathbf{r}'| \rightarrow \infty. \quad (16)$$

Here, $\hat{\phi}(\mathbf{r}) = \int d\mathbf{r}' v(|\mathbf{r} - \mathbf{r}'|) \hat{\rho}(\mathbf{r}')$ is the microscopic electric potential created at point \mathbf{r} by the system of charged particles, and the distance $|\mathbf{r} - \mathbf{r}'|$ has to be large compared to the microscopic scale represented by the correlation length of the short-ranged (exponentially decaying) particle correlations. Since the microscopic electric field $\hat{\mathbf{E}}$ is given by $\hat{E}_{\mu}(\mathbf{r}) = -\partial_{\mu} \hat{\phi}(\mathbf{r})$ ($\mu = x, y, z$), the field-field correlation function is obtained from eq. (16) as

$$\beta \langle \hat{E}_{\mu}(\mathbf{r}) \hat{E}_{\nu}(\mathbf{r}') \rangle^T \sim \frac{3(\mathbf{r} - \mathbf{r}')_{\mu}(\mathbf{r} - \mathbf{r}')_{\nu} - \delta_{\mu\nu} |\mathbf{r} - \mathbf{r}'|^2}{|\mathbf{r} - \mathbf{r}'|^5}. \quad (17)$$

It is obvious that $\langle \hat{\mathbf{E}}(\mathbf{r}) \rangle = 0$ since the mean electric potential is a constant inside a conductor. However, the asymptotic formula (17) tells us that nonzero thermal fluctuations of the electric field must be present in the system for any particle density n . The generalization of the fluctuation results, obtained for the bulk, to inhomogeneous situations of the present interest, like conductors with boundaries, was made in ref. [23]. As soon as the two points \mathbf{r} and \mathbf{r}' are inside a conductor, asymptotic formulae (16) and (17) remain valid. When one of the points lies on the conductor boundary, the tangential components of the electric field at this point still fluctuate according to (17), while the discontinuity of the normal component across the surface is related to surface charge fluctuations. These fluctuation phenomena make the living conductors fundamentally different from the inert ones with tangential components of the electric field at a boundary identically set to zero, as in eq. (1). The Casimir force (12) can be retrieved through a Maxwell stress tensor computed from the electric-field fluctuations in the vacuum region [24].

As was already mentioned, our Coulomb fluid of classical charged particles with de Broglie hard cores can represent its quantum counterpart of pointlike charges at sufficiently large temperatures. It has been shown in [1] that the long-range charge correlations of the semi-classical regime do not spoil the classical limit (12). The high-temperature region of classical fluids is described exactly by the Debye-Hückel (DH) theory. Rigorous conditions, under which the DH approximation gives the exact leading correction to the ideal gas, were the subject of many studies in the past; for a short historical review, see *e.g.* [20]. These conditions arise naturally in a renormalized Mayer diagrammatic expansion for statistical quantities [25, 26]. In terms of the mean interparticle distance a and the inverse Debye length κ (κ^{-1} is the correlation length of particles in the DH regime), defined by

$$\frac{4\pi a^3}{3} = \frac{1}{n}, \quad \kappa^2 = 4\pi\beta e^2 \sum_{\alpha} Z_{\alpha}^2 n_{\alpha}, \quad (18)$$

the DH scaling regime is given by (see eq. (11) of ref. [26])

$$\left(\frac{\lambda}{a}\right)^3 \ll \frac{1}{2}\kappa\beta e^2 \ll 1, \quad (19)$$

where λ represents the “typical”, in our case de Broglie, hard-core radius of particles. Supposing in what follows for the sake of simplicity that $\sum_{\alpha} Z_{\alpha}^2 n_{\alpha}/n$ is of order of unity and omitting irrelevant numerical factors, these inequalities can be rewritten in a more transparent form:

$$a_0 \ll a \ll \kappa^{-1}, \quad (20)$$

where $a_0 \sim \hbar^2/(me^2)$ with m being the “typical” particle mass. The lightest of the charges are the electrons for which the quantum microscopic scale a_0 attains its maximum value, equal to the Bohr radius $\sim 10^{-10}$ m. The first “Bohr” inequality in (20) is a quantum upper bound for possible values of particle densities. Since the Bohr radius is small, very dense Coulomb fluids with $a \sim 10^{-8}$ m are allowed; we shall refer to them as “living ideal conductors”. The second inequality combines both the particle density n and the temperature parameter βe^2 ; for a fixed particle density allowed by the first inequality, there always exists a sufficiently high temperature above which this inequality is fulfilled.

Let our classical Coulomb fluid model the conductor slabs in the Casimir geometry. The characteristic correlation length of the particle system is assumed to be much smaller than the macroscopic vacuum distance d between the conductor walls,

$$\kappa^{-1} \ll d. \quad (21)$$

Then, the t -parameter (5) is of the form $A(\kappa d)/(\kappa a)^3$ with A of the order of $12\pi e^2/(\hbar c)$, *i.e.* unity. The scale hierarchy (20) and (21) is thus fully consistent with the classical and nonrelativistic limit of interest $t \rightarrow \infty$. To summarize: as soon as the scaling length regimes (20) and (21) apply, the quantum system of charged particles coupled to electromagnetic radiation at equilibrium can be represented in terms of its classical pure-Coulomb fluid counterpart, decoupled from radiation and treated within the DH theory. The Casimir force originates exclusively from the thermal fluctuations in the conducting walls modelled by this classical Coulomb fluid.

The Casimir problem of microscopic Coulomb fluids was solved by using an inhomogeneous version of the DH theory in two recent papers: the work [24] dealt also with more complex physical situations; the study [27] went beyond the DH theory. In the DH theory, the large- κd expansion of the Casimir force was obtained in the form [24]

$$f^T(d) = -\frac{\zeta(3)}{8\pi\beta d^3} \left\{ 1 - \frac{6}{(\kappa d)} + \mathcal{O}\left(\frac{1}{(\kappa d)^2}\right) \right\}. \quad (22)$$

The leading universal term is identical to the Lifshitz result (12). The subleading correction term is non-universal and depends on the composition of the Coulomb fluid via κ . Even for a very dense Coulomb fluid with the mean interparticle distance $a \sim 10^{-8}$ m (living ideal conductor), there exists a sufficiently high temperature and a sufficiently large distance between slabs above which the required length scale hierarchy $a \ll \kappa^{-1} \ll d$ takes place and the correction term is negligibly small in comparison with the leading one.

In conclusion, the mathematical model of inert ideal-conductor walls is based on the zero boundary conditions for the tangential components of the electric field (1). This seems to be in agreement with experimental results at zero temperature, and perhaps also at sufficiently

small temperatures. Why the quantum ground-state fluctuations in the real walls seem to play no role is an open problem. At high temperatures, fluctuations of the electric field prevail in the bulk and at the surface of the particle system, even in the high-density limit (living ideal conductor), which makes the enforcement of the inert boundary conditions inadequate. Within the hierarchy of length scales (20) and (21), the high-temperature Casimir force was shown to be entirely determined by the thermal fluctuations of the conducting walls, modelled microscopically by classical Coulomb fluids in the Debye-Hückel regime.

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