Modelling synchronized flow at highway bottlenecks

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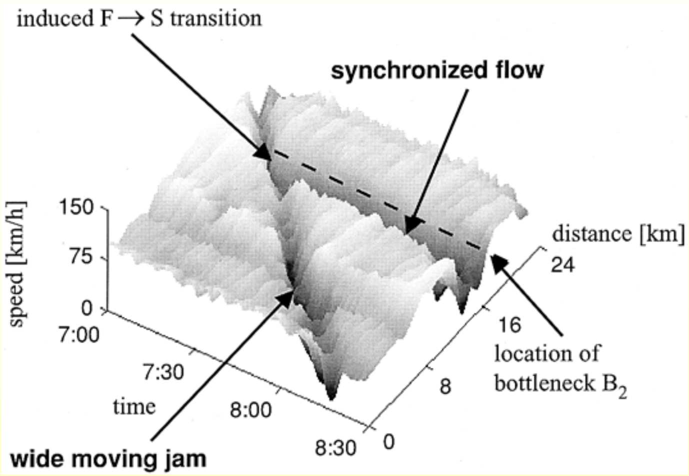
- Motivation
- Balanced vehicular traffic
- Modelling highway bottlenecks

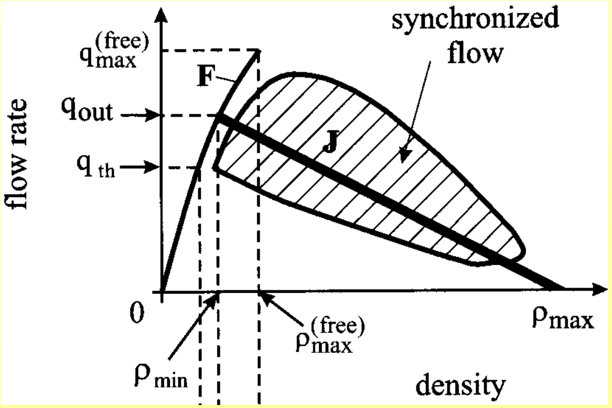
Motivation

Spatio-temporal dynamics of traffic flows

- Kerner's three phases of traffic flow:
  - **free flow**
    - high velocities and low densities
  - **wide moving jams**
    - very small velocities and large densities
    - robust, travel through bottlenecks
    - constant propagation velocity of the downstream jam front
    - outflow nearly constant
  - **synchronized flow**
    - small velocities and large densities
    - wide scattering in the fundamental diagram
    - often localized at a bottleneck
- Capacity drop at highway bottlenecks:
  - outflow from synchronized flow region below maximum capacity of free flow



B. Kerner, The Physics of Traffic (2004)

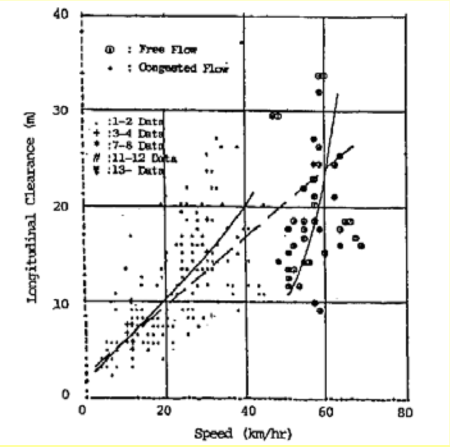


Aw, Rascle, Greenberg model

- Model by A. Aw and M. Rascle (SIAP 2000), J. Greenberg (SIAP 2001)
  - M. Zhang (Transportation Research B, 2002)
- deterministic, effective one-lane-model
  - continuity equation: $$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0$$
    momentum equation: $$\frac{\partial (\rho (v - u(\rho)))}{\partial t} + \frac{\partial (\rho v (v - u(\rho)))}{\partial x} = \frac{\rho (u(\rho) - v)}{T}$$

ρ: vehicle density
v: velocity
u(ρ): equilibrium velocity
T>0: relaxation time

  - hyperbolic system of balance laws
    - anisotropic condition
  - motivation for an extension:
    - multi-valued fundamental diagram
    - instabilities
    - instantaneous reaction to the current traffic situation



M. Koshi et al (1983)

Balanced vehicular traffic

Balanced vehicular traffic

- Extended Aw-Rascle-Greenberg model:

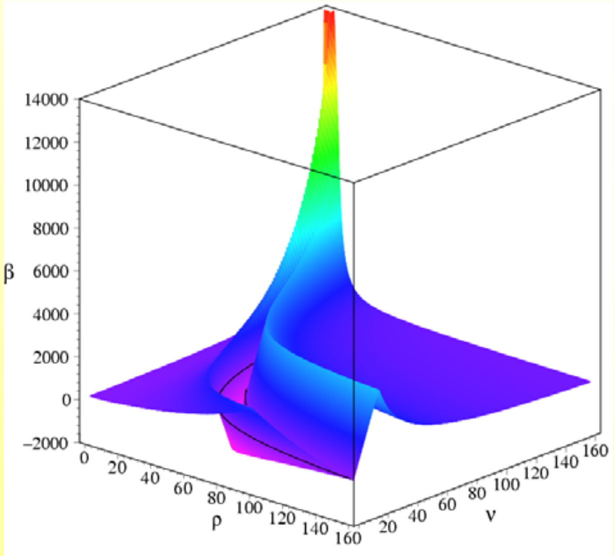
$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0$$

continuity equation

$$\frac{\partial (\rho (v - u(\rho)))}{\partial t} + \frac{\partial (\rho v (v - u(\rho)))}{\partial x} = \beta(\rho, v) \rho (u(\rho) - v)$$

momentum equation

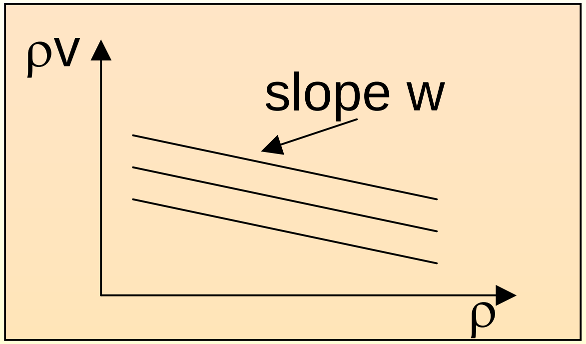
- Characteristic speeds
  - $\lambda_1 = v + \rho u'(\rho) \leq v$
  - $\lambda_2 = v$
- Effective relaxation coefficients: $\beta(\rho,v)$
  - ARG: constant, inverse relaxation time
  - here: function of density ρ and velocity v
- Papers:
  - F. Siebel, W. Mauser, *SIAP 66, 1150 (2006)*
  - F. Siebel, W. Mauser, PRE 73, 066108 (2006)



Steady state solutions

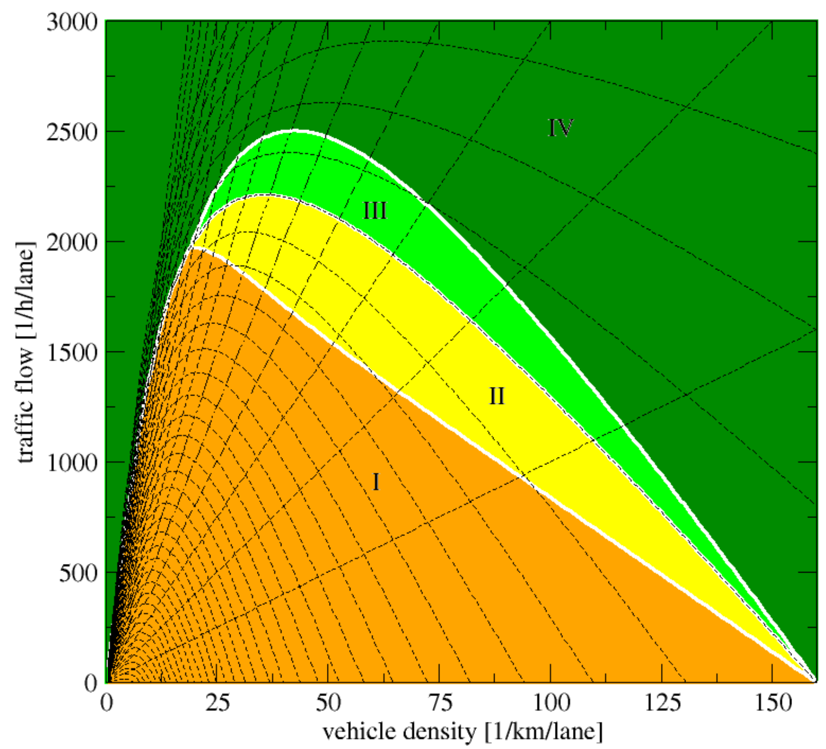
- Steady state solutions:
  - moving coordinate system: z = x - wt
  - continuity equation:
    - steady state solutions lie on straight lines in the fundamental diagram
  - *momentum* equation:
    - trivial solutions:
      - **equilibrium velocity** v = u(ρ)
      - two additional solution branches, the **zeros** of the effective relaxation coefficient β(ρ,v)
        - **jam line**
        - **high flow branch**
    - non-trivial solutions:
      - monotonous solutions linking two trivial branches
      - solutions of the ODE

$$(\lambda_1 - w)\frac{dv}{dz} = \beta(\rho, v)\big(u(\rho) - v\big), \quad \rho = \frac{q}{v - w}, \quad q = const$$



Steady state solutions

- Trivial steady-state solutions:
  - white lines
- Non-trivial steady-state solutions
  - regions II and III
- Characteristic curves
  - black dashed lines
    - $\lambda_1 = v + \rho u'(\rho) \leq v$
    - $\lambda_2 = v$



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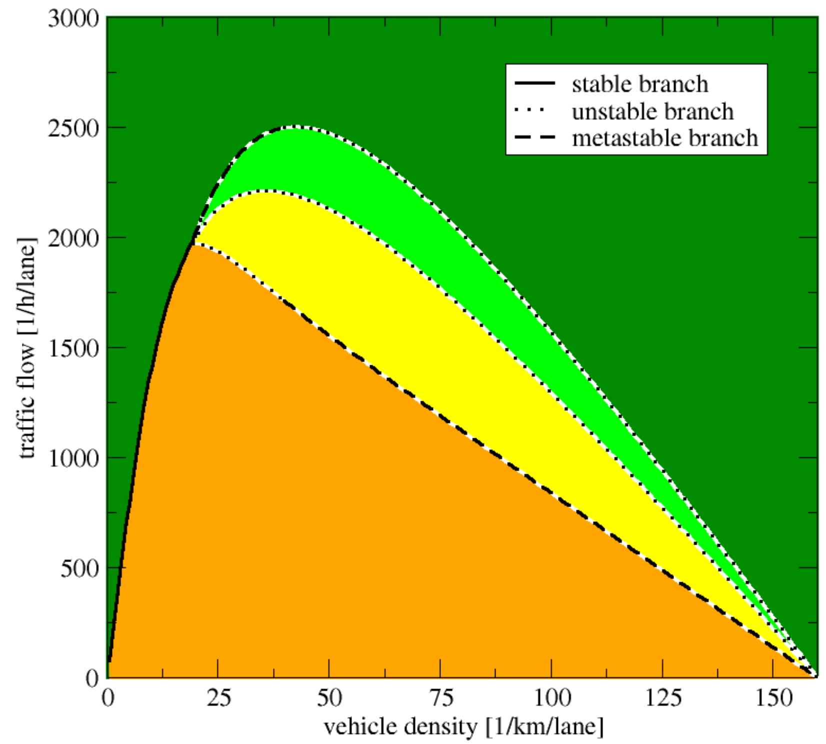
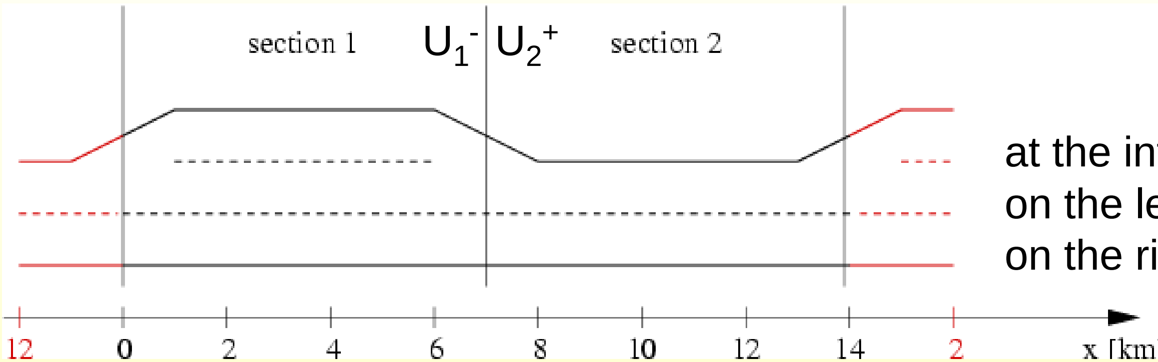
    - $\lambda_1 = v + \rho u'(\rho) \leq v$
    - $\lambda_2 = v$
- Stability:
  - subcharacteristic condition
- Traffic phases:
  - free flow
  - wide moving jams



Modelling highway bottlenecks

Lane reduction

- Reduction from 3 to 2 lanes:

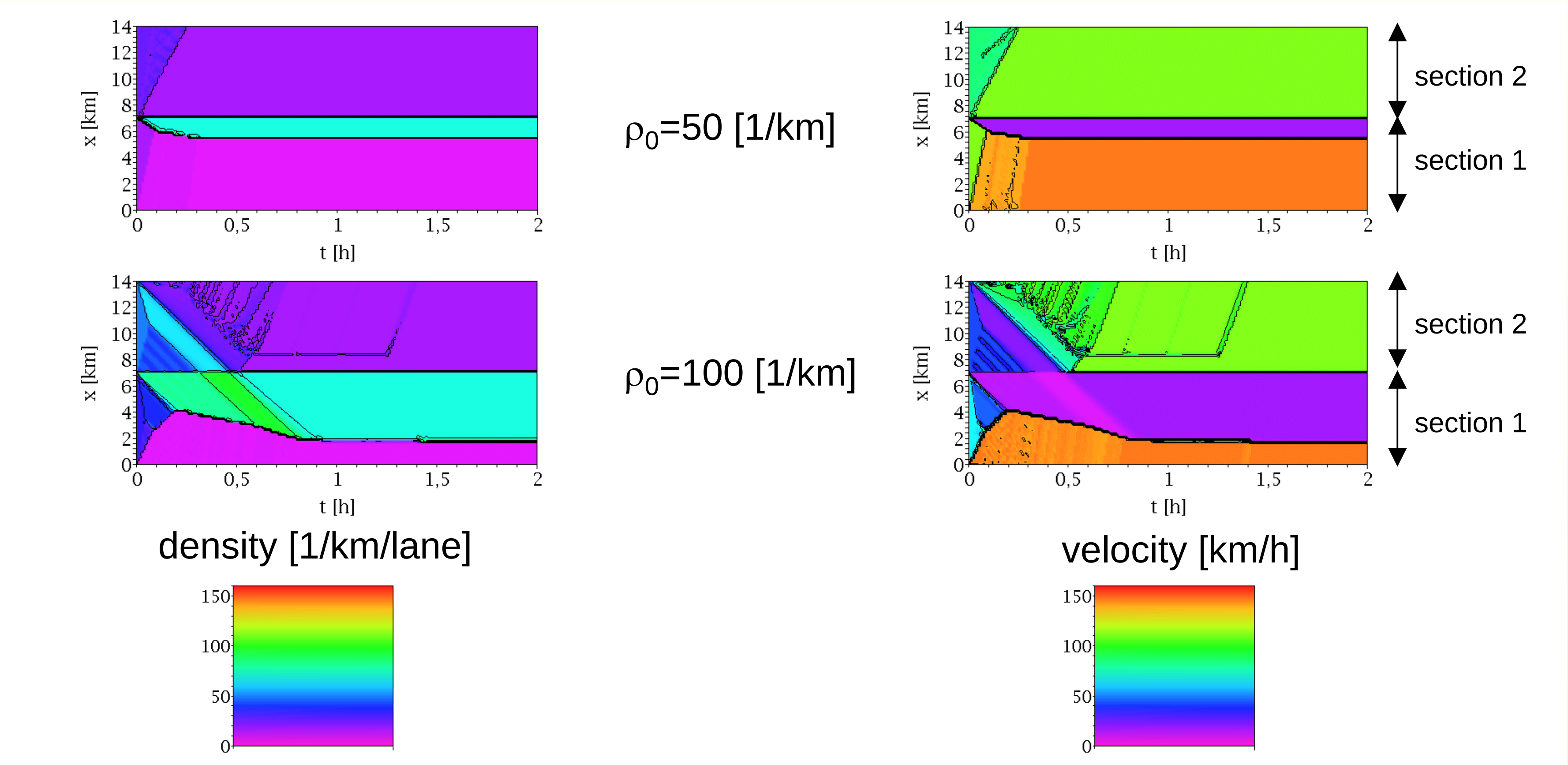


at the interface:
on the left in section 1: $U_1^- = (\rho_1^-, v_1^-)$
on the right in section 2: $U_2^+ = (\rho_2^+, v_2^+)$

- Coupling conditions at the interface: Riemann problem for principal part
  - conservation of vehicle number: $\rho_1^- v_1^- = \rho_2^+ v_2^+ = q$
  - "conservation" of the *momentum*: $\rho_1^- v_1^- (v_1^- - u_1(\rho_1^-)) = \rho_2^+ v_2^+ (v_2^+ - u_2(\rho_2^+))$
  - maximization of the total flux q: determined from the demand and supply
  - coupling conditions: Haut, Bastin (2005), Garavello, Piccoli (2006), Herty, Rascle (2006), Herty, Moutari, Rascle (2006)
  - applicable to a general junction with arbitrary number of incoming and outgoing roads

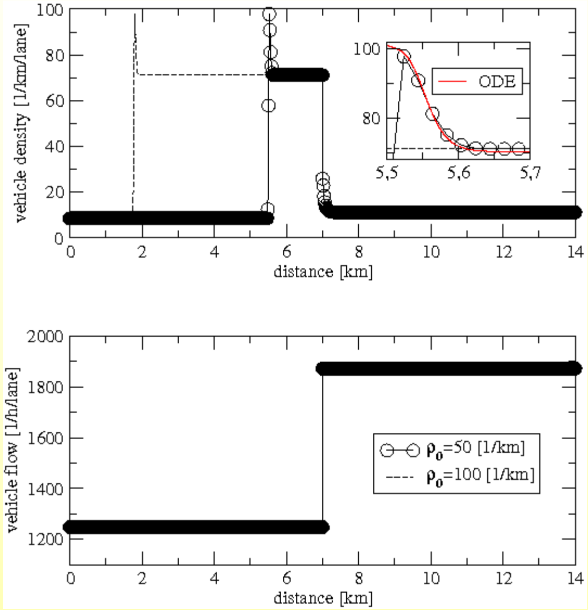
Lane reduction

- Numerical simulations of constant initial data ρ_0 in equilibrium $v = u(\rho_0)$:

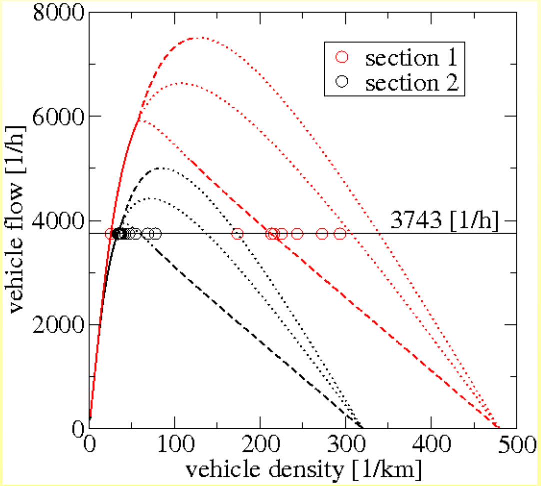


Lane reduction

- Static solutions at the bottleneck:



  - solutions with constant flow
  - non-constant curve sections for density profile correspond to non-trivial steady-state solutions
  - interfaces between sections determined by the conservation of mass and *momentum*
  - von Neumann state downstream of the shock
  - **Capacity drop**: flow value below maximum in section 2 (about 1900 [1/h/lane])



- determined by the crossing of the static solutions with the jam line (low-flow equilibrium branch)
- consequence of the source term
- on/off-ramps: http://arxiv.org/abs/physics/0609237

Conclusion

- Balanced vehicular traffic model
  - describes traffic flow as a hyperbolic system of balance laws
  - capable of reproducing
    - multi-valued fundamental diagrams
    - metastability of free flow at the onset of instabilities
    - synchronized flow and wide moving jams
    - capacity drop
  - these aspects of traffic flows can be reproduced **without** modelling
    - lane changes
    - different driver characteristics and vehicle types (trucks)
    - stochastic elements
  - importance of nonlinear instabilities