Macroscopic relaxation after on-ramps in real data and in cellular automata simulations

Cécile Appert-Rolland (corresponding author)[a,b], Jérémie Du Boisberranger[a,b]

[a] *Univ. Paris-Sud, Laboratoire de Physique Théorique, UMR8627*
Bâtiment 210, Orsay F-91405, France
[b] *CNRS, Orsay F-91405, France*
Cecile.Appert-Rolland@th.u-psud.fr

Abstract

This paper presents a methodology for the testing of lane changing rules in traffic simulation models. The idea is to use simple macroscopic data to test microscopic rules. More precisely, the relaxation of the lane flow rates after an on-ramp on a multilane highway is taken as an integrated signature of the lane-changing efficiency.

Our approach is two-fold. First the characteristic length scale on which multilane traffic relaxes after a perturbation (here an on-ramp) is measured under various conditions on real data. It is shown to be of the order of a few hundred meters. In the free flow regime, this relaxation length was found to increase with the occupancy when the perturbation due to the on-ramp is weak, while it becomes independent from the occupancy for strong on-ramp perturbations. By contrast, in the congested regime, the relaxation length was found to decrease with increasing occupancy.

The comparison of experimental observations with simulation results allows to test indirectly the relevance of the lane changing rules used in the model. Here we apply this approach to cellular automata based simulations. We evidence and discuss several shortcomings of the numerical method.

Keywords: relaxation length, on-ramp, lane changing, inductive loop data, cellular automata, highway traffic

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1. Introduction

In the past decades, continuous effort has been devoted to the study of longitudinal interactions between the vehicles, partly because it was needed in order to develop realistic car-following models, partly because the measurement means that were available at this time were giving the corresponding informations. It is technically more difficult to obtain information about lateral interactions, and lane changing behaviors. However, the development of new tools and the increase of storage capacities now allows to have access to more refined information, and in particular to study the interactions between the vehicles of different lanes on a highway (Appert-Rolland, 2009). This possibility is all the more interesting since lane changes are expected to play an important role in security issues, or in the more refined description of traffic states - such as synchronized traffic states. Moreover, the numerous lane changes that are required in the vicinity of merging areas play an important role in the formation of localized bottlenecks and in capacity drops (Cassidy and Rudjanakanoknad, 2005; Banks, 2006; Laval and Daganzo, 2006; Laval et al., 2007; Lertworawanich and Elefteriadou, 2007; Duret et al., 2009, 2010b).

Lane changing rules have been introduced in various types of traffic models, in a way that is most often good-sense based, but not always fully verified. Thus there is a need for a better understanding of lane changes, and of their consequences on the structure of the flow, as this was explored for example by Kerner and Klenov (2009), or on the capacity of infrastructures.

Lateral interactions can be studied from a microscopic point of view. The NGSIM project, for example, has yield some detailed informations about the trajectories of vehicles that have been extensively studied since then (in particular by Leclercq et al. (2007),Toledo and Zohar (2007), and Thiemann et al. (2008)). However, it is always a quite heavy task to extract and analyze microscopic data.

Here, we propose a complementary approach, that allows to test microscopic rules based on macroscopic observations. Our proposal can be seen as a follow-up of the works by Nagel et al. (1998), Knospe et al. (2002), and Duret (2010), for which the measurement of the macroscopic lane flow distribution on homogeneous stretches of multilane freeways was used as a test for microscopic lane changing rules.

We propose that it is possible to go one step further in this testing process, by considering the macroscopic response of a highway to a local perturbation (namely an on-ramp). The system that we consider is not homogeneous in space anymore, as the presence of the on-ramp breaks the translational invariance. The macroscopic quantities that we are considering are the lane flow distribution, or the lane flow rates.

In the following, we shall say that the system has reached an equilibrium between the lanes at a given position if the lane flow distribution at this position is the same as it would be on a long homogeneous freeway section with the same total flow rate. We assume that the on-ramp that we are considering is isolated enough, so that far enough upstream and downstream, an equilibrium between the lanes is reached. The upstream and downstream equilibriums are expected to be different, as the total flow rate is different before and after the on-ramp. Obviously, at the level of the on-ramp, the lane flows are not equilibrated, as the incoming on-ramp flow perturbs the upstream equilibrium.

The response of the system to the perturbation induced by the on-ramp can be characterized, first, by the amplitude of the lane flow variations at the level of the on-ramp, and second, by the relaxation of the lane flow distribution towards a new downstream equilibrium between the lanes. These two features are expected to depend, among others, on the on-ramp incoming flow rate, or on the traffic state.

Here we shall focus on the relaxation process, and more precisely on the characteristic length on which relaxation occurs. The relaxation length results from the combined behavior of the vehicles, i.e. it somehow integrates the various lane changes that lead to a new equilibrium between the lanes. Thus, measuring whether a simulation model reproduces well these relaxation measurements is an indirect test of the validity of its lane changing dynamics. It is not always necessary to have fully realistic lane changing rules in the simulation models, but the rules should at least retain some minimal features in order to get realistic macroscopic behavior. The definition of the relaxation length will be made more precise in section 2.

The methodology that we want to present in this paper can be divided into two steps - hence the two main sections of our paper. First we want to extract some knowledge about relaxation lengths from real data. We shall present in section 3 some data analysis that was performed from inductive loop data obtained on a three-lane highway near Paris (France), respectively on two sites (subsections 3.1 and 3.2). We have measured the relaxation length after on-ramps for several traffic states and several flow levels.

In a second part (section 4), these lengths are compared with those obtained in numerical simulations of a microscopic model. In this paper, we shall exemplify this approach on a cellular automata model. Nagel et al. (1998) and Knospe et al. (2002) have characterized the requirements for having a realistic lane flow distribution on homogeneous freeway stretches with cellular automata models, and in particular for observing the well-known phenomenon of density inversion. We shall show that the on-ramp test is more sensitive, and is able to point out some limitations of the model that were not visible by testing homogeneous conditions.

2. Definition of the relaxation length

Before presenting experimental or numerical measurements of the relaxation length, we define it in this section. Actually several definitions are possible, which we shall discuss now. But only the first one will be used in this paper.

It should be first noticed that we could consider the relaxation of several quantities: absolute flows or occupancies, or relative flows or occupancies (i.e. fractions), or velocities... Of course all these quantities are related, so that they should give the same order of magnitude for relaxation distances. Still, the shape of the relaxation profiles may be different. We chose to focus on absolute values of the lane flows, as these are directly related to the rates of lane changes.

The first definition, that will be used throughout the paper, assumes that the profile for the flow per lane (i.e. the flow per lane as a function of the spatial coordinate along the road) has an exponential shape. We shall see later that this is a good approximation only for the right lane. Still, this is the definition that we shall use in this paper and we shall see below how it could be easily generalized.

Definition 1. *If the lane flow profile can be fitted by an exponential*

$$A_0 \exp(-x/\xi) + A_1, \tag{1}$$

then the characteristic length ξ *is called the characteristic length of the relaxation process.*

For simplicity, ξ will be called the relaxation length in the following. However, it should be noted that with this definition, a distance much larger than the relaxation length is needed to relax towards equilibrium. The relaxation length indicates here only the distance needed for the perturbation amplitude to be divided by $e = 2.72$.

Another question is from where the distance x should be measured. The origin of distances is supposed to be at the on-ramp, but the merging area of the on-ramp actually has a rather large spatial extension (200 meters for the data presented in this paper). Here, we took as the origin for x the beginning of the merging area (for reasons that will be explained in the next section).

We could imagine to have more general definitions for the relaxation length, that could be valid without any assumption on the shape of the flow profile. An example could be the following.

Definition 2. *The relaxation length λ_θ is the distance needed for the amplitude of the perturbation to be divided by θ*

where θ could be equal to 10 for example. When the relaxation profile is exponential, there is a direct link between ξ and λ_θ, through the relation

$$\lambda_\theta = \xi \ln \theta. \tag{2}$$

Actually our methodology, based on the comparison of the relaxation length in experiments and numerical simulations, can be generalized for any choice of the relaxation length definition, as long as the same definition is used for the comparison.

In the remaining of the paper, the expression 'relaxation length' will always refer to ξ.

3. Experimental response to an on-ramp

The data that we have used were obtained from inductive loops placed approximately every 600 meters[1], on each of the three lanes of highway A6, going outward from Paris (France). Flow rate, velocity, and occupancy are aggregated over 6 minutes time intervals. We analyzed two sets of data.

- The first one corresponds to the surroundings of an on-ramp coming from a local road (D25) - a schematic representation of the geometry is given in figure 1. For this site (which will be called site A6/D25 in the

[1] Inter-loop distances are ranging from 470 to 670 meters, with an average at 572 meters.

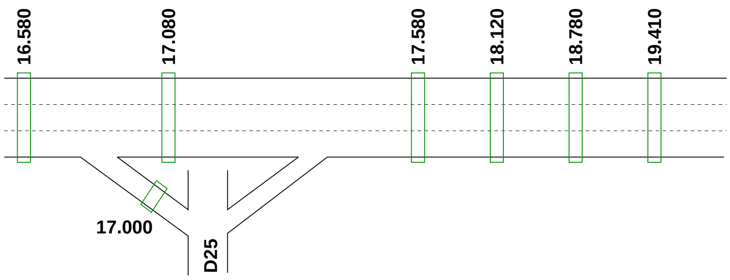

Figure 1: Site A6/D25. Schematic representation of the highway A6 and the on-ramp from D25. Inductive loops locations are indicated by rectangles. Vehicles are driving from left to right. The scale is not respected.

remaining), we have selected only the time intervals for which the on-ramp flow rate (obtained from the difference between the downstream and upstream flow rates) was significant compared with the flow rate on the highway. As a lower boundary for the on-ramp activity we have chosen that the flow rate on the on-ramp must be larger than 12.5 % of the total flow rate. The remaining data corresponds to 160 000 vehicles, and an average on-ramp flow rate equal to 14.7 % of the total downstream flow rate (equal on average to 4600 veh/hour).

At this site the downstream fundamental diagram for the flow on the highway indicates a transition from free flow to congested traffic for an occupancy around 16-18% on the right lane (see Fig. 2). Most of the data points are in the free-flow regime.

The speed limit is 90 km/h for trucks with a weight greater than 3.5 metric tons, and 110 km/h for other vehicles.

- The second site (called site A6/A10 in this paper) corresponds to the exchange ramps between two highways coming from Paris (A6a and A6b/A10), as indicated in figure 3, and we are interested in the relaxation of traffic on A6a after the on-ramp.

  Although there were no data available directly behind the on-ramp for the left and the middle lane, we decided to analyze the data from this site as well, since the activity of the on-ramp is considerably higher than for the first site. As a result we observe much more frequently

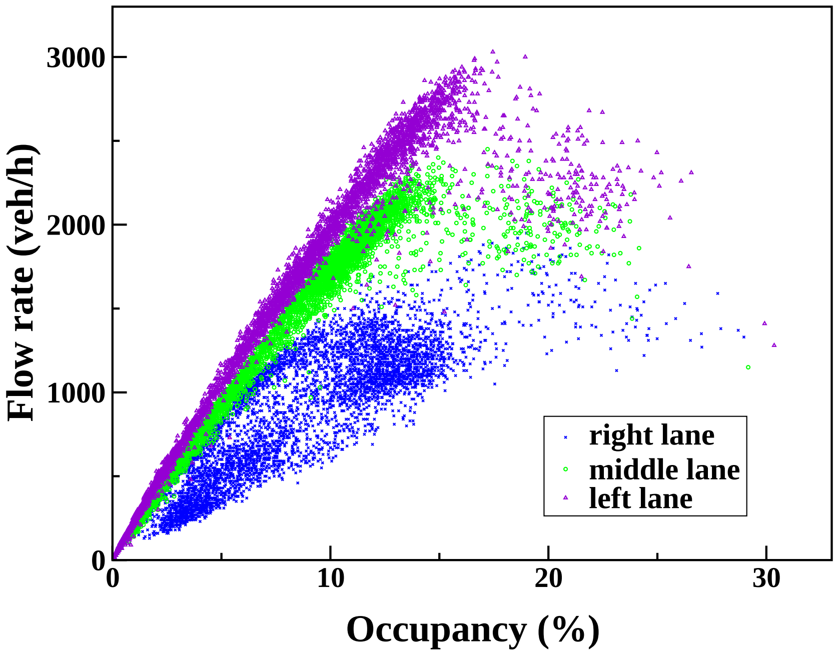

Figure 2: Fundamental diagram in site A6/D25, at km 18.78 (downstream the on-ramp), based on 6mn averaged data collected during one day.

congested traffic and extremely high values of the occupancy[2] (see Fig. 4). On average, the on-ramp flow rate represents 40% of the total downstream flow rate. The data were including 1.657.340 vehicles in total, among which 1.411.740 for free flow and 245.594 for congested traffic. The speed limit is 90 km/h.

3.1. Analysis of site A6/D25 data

We shall first consider site A6/D25 and focus on the free flow regime.

Fig. 5 shows an example of lane flow profiles after the on-ramp, for a given range of the total flow rate, in the free-flow regime. On the right lane, relaxation starts immediately, and the flow rate relaxes exponentially towards a new equilibrium value. Relaxation on the left lane is not exponential: it

[2]We observed occupancies up to ∼50%, while the maximal value was restricted to 25% for the first data set. Note that each data item represents an average over 6 minutes interval.

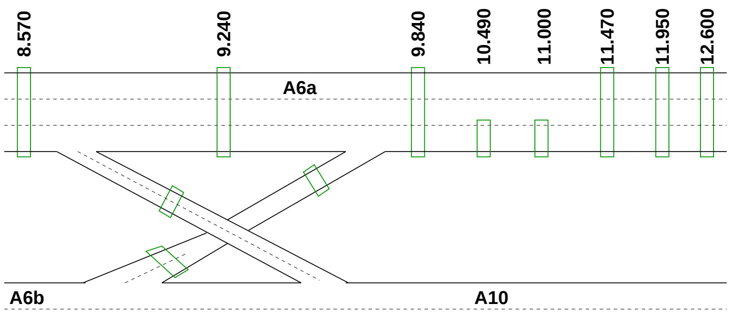

Figure 3: Site A6/A10. Schematic representation of the exchange ramps between highways A6a and A6b and of the inductive loops locations (indicated by rectangles). Vehicles are driving from left to right. The scale is not respected. Only the left lane of highway A6b/A10 is represented.

occurs with a delay, and as a result there is an inflection point in the flow profile.

As the total flow rate is conserved along the highway, the flow variations on each lane are directly related to the lane changing rates. Thus, from the profile of Fig. 5, it appears that most lane exchanges between the right and middle lane occur at the level of the on-ramp or shortly after, while most lane exchanges between the middle and left lane occur with a delay - as should be expected. The flow profile on the middle lane results from these two phenomena, which overlap.

• **Relaxation length measurements**

We assumed that the measurements of the first detector next to the on-ramp in upstream direction (km 17.080) are also valid at the beginning of the on-ramp, i.e. at km 17.330. Indeed, as the on-ramp follows an off-ramp, we expect the drivers who want to anticipate their lane change to do it quite early. This assumption is in agreement with direct observations of the traffic states at site A6/A10 (Louis, 2010), and will be examined further in future work. As a consequence of this assumption, we consider that the flow rate on the right lane at the beginning of the on-ramp merging is equal to the sum of the flow rate on the right lane before the on-ramp plus the flow rate coming from the on-ramp. To be consistent with this assumption, it is natural to measure the distance from the on-ramp actually from the beginning of the merging area. It would be interesting in the future to obtain data within the merging region, in order to check which choice is the most relevant.

In the free flow regime, we find that the relaxation length on the first lane

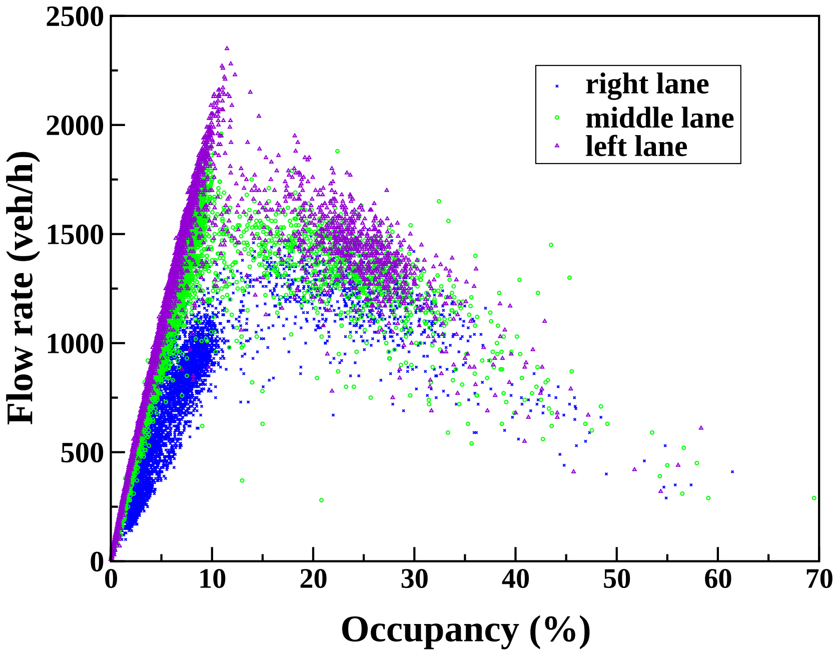

Figure 4: Fundamental diagram in site A6/A10, at km 12.6 (downstream the on-ramp), based on 6mn averaged data.

increases with an increasing flow rate (see Fig. 6). Note that, for each bar of this figure, the input flow from the on-ramp represents always more or less the same ratio of the total flow, as indicated in table 1, so that the effect that we observe cannot be explained by an increasing fraction coming from the on-ramp. The effect is even more visible if we plot the relaxation length as a function of the occupancy (see Fig. 7). Actually this can be easily understood: the number of gaps large enough to allow for a lane change decreases rapidly when the density becomes large.

Note that, for the largest relaxation lengths, we were not able to see the complete relaxation towards constant values behind km=19.3. Indeed, long relaxation length measurements are valid only if the highway portion that is considered is homogeneous. Here, the velocity of the flow increases on the right lane after km=19.3. This is due to the fact that before that point, trucks are not allowed to pass other vehicles and are limited to 90 km/h, while the speed limit is 110 km/h for other vehicles; after that point, trucks are allowed to behave as other vehicles. Besides, the influence of the next intersection, located at km 20, cannot be excluded beyond that point.

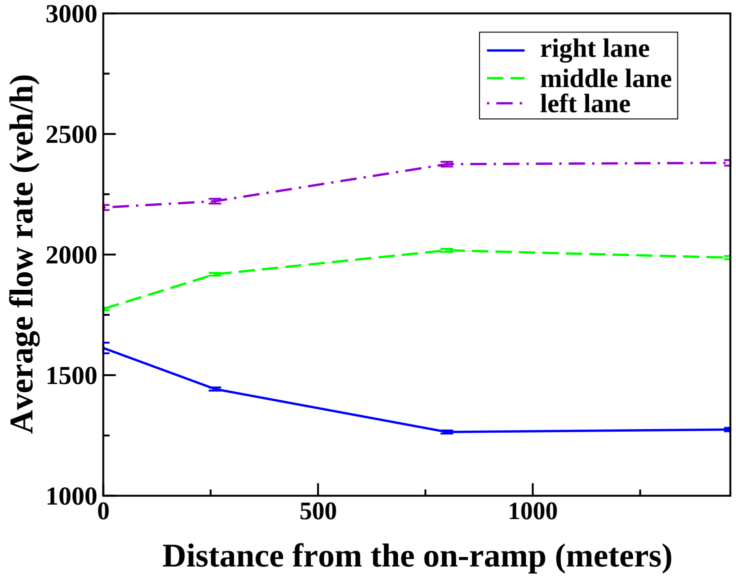

Figure 5: Flow rates on the three lanes, for a total flow rate between 5000 and 6000 veh/h, and for an occupancy less than 16 % (free flow regime). The error bars are twice the standard error on the mean, and are so small that they can hardly be distinguished from the lines. Data from site A6/D25.

min flow rate	max flow rate	ratio on-ramp / total flow rate
0	4000	15.7 %
4000	5000	14.0 %
5000	6000	14.2 %
6000	6700	14.6 %

Table 1: Percentage of the total downstream flow coming from the on-ramp, for the flow intervals considered in figure 6. This percentage is more or less constant, except for the first point slightly higher. Data obtained in site A6/D25.

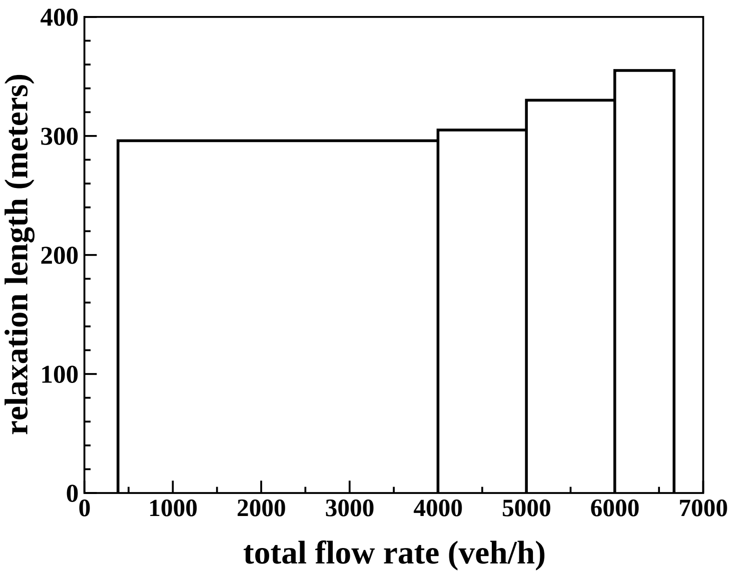

Figure 6: Relaxation lengths in the free flow regime, as a function of the total flow rate. Relaxation lengths are measured in site A6/D25 on the right lane, for an occupancy less than 16 % (free flow regime), and for various ranges of the total downstream flow rate (resp. less than 4000, between 4000 and 5000, between 5000 and 6000, and greater than 6000 veh/h). Each measurement involves from 17 000 to 58 000 vehicles. The percentage of the downstream flow rate coming from the on-ramp is almost constant and equal on average to 14.7 %.

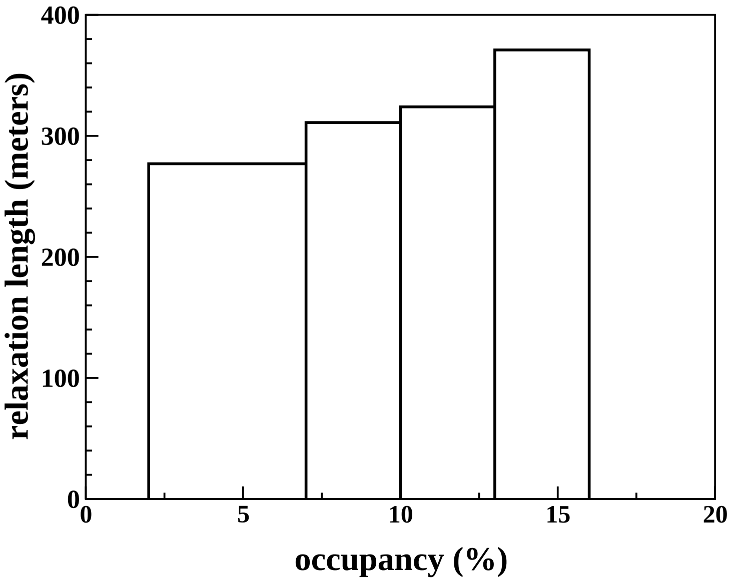

Figure 7: Relaxation lengths in the free flow regime, as a function of the occupancy. Relaxation lengths are measured in site A6/D25 on the right lane, for various ranges of the occupancy measured on the right lane just after the on-ramp. Each measurement involves from 120 000 to 400 000 vehicles. The percentage of the total downstream flow rate coming from the on-ramp is almost constant and equal on average to 14.7 %.

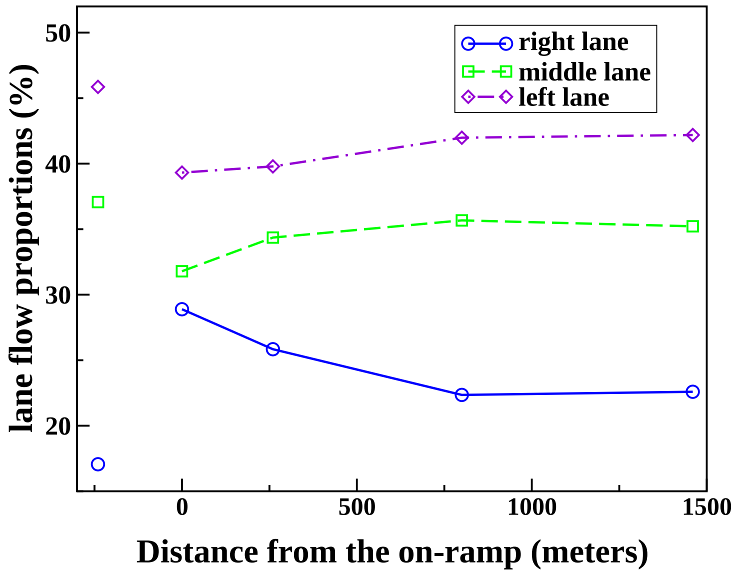

Figure 8: Lane flow proportions as a function of the distance to the on-ramp, for a total downstream flow rate between 5000 and 6000 veh/h, and for an occupancy less than 16 % (free flow regime). Data from site A6/D25. This figure corresponds to the same data set as the one used in figure 5. Here we have added the lane flow proportions before the on-ramp, for comparison.

Thus measurements cannot be extended too far downstream. As, besides, the magnetic loops are a few hundred meters apart from each other, the values that we give for the relaxation lengths should be taken as orders of magnitudes rather than precise measurements. However, first we believe that the trends that we obtain are nevertheless significant, and second, we want here mainly to exemplify a procedure that can afterwards be reproduced with other data sets.

The possible influence on the relaxation process of slow vehicles such as trucks, in particular when they are not allowed to change lane, would deserve further investigation. Indeed, they may trigger lane changes of other vehicles and thus reduce the relaxation lengths.

Instead of the absolute value of lane flow rates, we can also measure the lane flow proportions, as shown in figure 8. The relaxation process is also clearly visible on these variables. As expected, after the on-ramp, the system relaxes towards a lane flow distribution which is different from the lane flow distribution observed before the on-ramp. Indeed, the total flow is different

min occupancy	max occupancy	ratio on-ramp / total flow rate
0	3.5	33.5 %
3.5	5.5	36.6 %
5.5	7.5	40.0 %
7.5	9.5	42.3 %

Table 2: Percentage of the total downstream flow coming from the on-ramp, for various occupancy intervals, and in the free flow regime. Variations are much larger than in tables 1 and 4. Data obtained in site A6/A10.

before and after the on-ramp, and the lane flow distribution is known to depend on the total flow rate (see for example (Duret et al., 2010a) for a data analysis of the lane flow distribution). Note however that here, the lane flow distribution before the on-ramp (measured at km 17.08) may not have reached its equilibrium value, as there is an off-ramp just before (as shown in Fig. 1).

In the remaining of the paper, we shall focus on lane flow rates rather than on lane flow proportions, because variations of lane flow rates between two positions can directly be interpreted in terms of a net number of lane changes that occurred between these two positions.

As seen on the fundamental diagram, no severe congestion is observed on this site, i.e. only congestion with a rather large flow rate were observed. A more systematic study of the congested state will be performed in the next section, for site A6/A10.

3.2. Analysis of site A6/A10 data

• **Free flow state measurements**

While at the A6/D25 site, the ratio of the on-ramp flow rate to the total downstream flow rate was always more or less constant, this is not the case here. Indeed, at the A6/A10 site, and in the free flow regime, the on-ramp flow rate increases more rapidly than the total flow rate, as shown on table 2. In order to avoid this effect, we selected only the 6mn-data for which the on-ramp flow rate represents between 37 and 43% of the total downstream flow rate. Then, on average, the percentage of the total downstream flow rate coming from the on-ramp is more or less constant (see table 3).

We measure the relaxation lengths for this subset (Fig. 9) and find, first, that the order of magnitude for the relaxation length ξ is the same as in the previous site. However, rather that an increase of the relaxation length

min flow rate	max flow rate	ratio on-ramp / total flow rate
0	800	39.7 %
800	1600	39.7 %
1600	2500	39.6 %
2500	3400	40.1 %
3400	5100	40.1 %

Table 3: Percentage of the average total downstream flow coming from the on-ramp, for the flow rate intervals considered in figure 9, in the free flow regime. Data have been selected such that in each 6mn interval, this percentage is between 37 and 43%. Then the mean value is rather constant for all flow intervals. Data obtained in site A6/A10.

min occupancy	max occupancy	ratio on-ramp / total flow rate
12	20	39.2 %
20	25	35.3 %
25	28	34.1 %
28	33	34.7 %

Table 4: Percentage of the total downstream flow coming from the on-ramp, for the occupancy intervals considered in figure 10, i.e. in the congested regime. Data obtained in site A6/A10.

with the total flow, we find it rather constant. This may be due to the fact that the perturbation due to the on-ramp is much larger here than in site A6/D25.

Another difference between the two sites is also that the velocity variance in the free flow phase, for a given occupancy, is much higher in site A6/A10 than in site A6/D25 - a feature that makes results in the A6/A10 site more difficult to interpret.

There would be a need to collect such measurements in various places in order to define more clearly which parameters determine the variations of the relaxation length in the free flow regime.

The analysis of the congested state in the A6/A10 site is more easy, in the sense that there is less variability in the traffic characteristics.

• **Congested state measurements**

On site A6/A10, in the congested state, the percentage of the total downstream flow rate that comes from the on-ramp is constant for all occupancy levels, as shown in table 4. Thus we could keep all data points for our analysis.

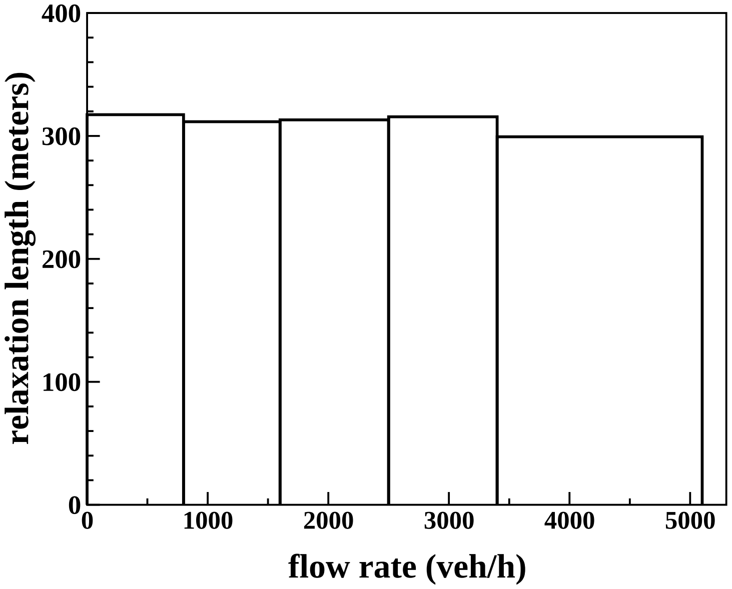

Figure 9: Relaxation lengths in the free flow regime, as a function of the total flow rate. Relaxation lengths are measured in site A6/A10 on the right lane, for an occupancy less than 9.5 %, and for various ranges of the total downstream flow rate. 6mn data were selected such that the percentage of the total downstream flow that comes from the on-ramp is between 37 and 43 %. The mean values for the on-ramp contribution are given in table 3.

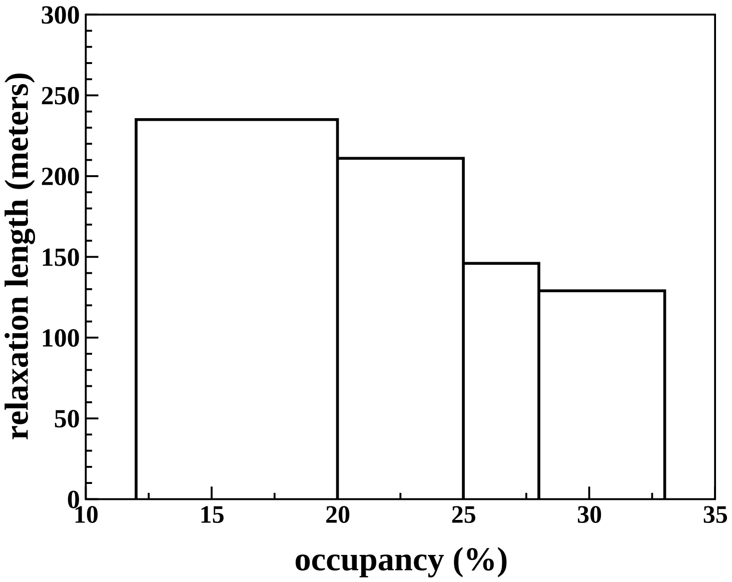

Figure 10: Relaxation lengths in the congested regime, as a function of the occupancy. Relaxation lengths are measured in site A6/A10 on the right lane. Each measurement involves from 13 600 to 18 200 vehicles. Measurements at the A6/A10 merging section, for an almost constant ratio on-ramp / downstream flow rate.

A large range of congested states can be observed, with pronounced velocity reductions. This allows us to explore how the relaxation length varies with the occupancy when traffic is congested. It turns out that, while the relaxation length was found to be constant or *increasing* with increased total flow rate in the free flow regime, it *decreases* with the occupancy in the congested regime (see Fig. 10). We can understand this feature by noticing that, in the congested traffic phase, the mean velocity decreases with the occupancy, a fact that favors a rearrangement between the lanes on shorter distances.

It can be noticed also that, while the transition to congested traffic was occurring for an occupancy of order 16% in site A6/D25, it occurs for a much lower occupancy (around 12%) here. This observation underlines the importance of lane-changes for the local capacity of a highway. The frequent lane-changes destabilize high flow states.

4. Response of a cellular automaton to an on-ramp perturbation

Cellular automata have been recognized as useful tools for traffic modeling since the pioneering work of Nagel and Schreckenberg (1992). This first model was already able to simulate free flow and stop and go traffic. On the other hand, dynamics were implying infinite braking capability, due to a lack of anticipation, and no metastability was included. Since then, more advanced variants have been proposed which overcome these difficulties. For example, the model by Knospe et al. (2000) includes anticipation (in particular through the use of brake lights), implements more realistic acceleration and deceleration values, and prescribes a lower acceleration for cars which came to a nearly stop in order to introduce metastability. As a result, the model is able to reproduce synchronized traffic, meaning that it is possible to stabilize high flow rates at high occupancies. This model has been quite well validated for one-lane traffic (Knospe et al., 2004). In particular, it reproduces realistic velocity-time headway relations.

Less is known about lane changing rules and their influence. Various lane changing rules have been proposed (Knospe et al., 2002). Nagel et al. (1998) and Knospe et al. (2002) have studied the impact of microscopic rules on the macroscopic behavior of the flow (see also (Wagner et al., 1997)), but only on homogeneous parts of the highway[3]. Here we propose to test the microscopic mechanisms related to lane changes through the space dependent macroscopic response to a localized perturbation (namely the on-ramp).

The model that we have used to simulate a portion of 3-lanes highway is based on the aforementioned Knospe et al. (2000) cellular automaton rules, with the lane changing rules described by Knospe et al. (2002). A lane change occurs if two criteria are met, an incentive criterium and a safety criterium. The incentive criterium describes under which conditions the vehicle tries to change lane. A lane change is tempted if the time headway of the vehicle is less than one time step (1 second in our simulations). We used asymmetric rules, which are appropriate for French traffic: a left to right lane change is also tempted if the time headway to the leader on the right target lane is greater than 3 seconds. This last rule induces a right-lane preference.

The safety criterium checks that the gap with the new leader and follower after lane change will be sufficient, not only to avoid collisions, but also to

[3]The test performed by Nagel et al. (1998) and Knospe et al. (2002) is somehow similar to the one performed by Duret (2010) for the Newell's car-following model.

minimize the interaction. In order to evaluate the gap with the leader on the target lane, it is important to estimate a lower bound for the velocity of the leader at the next time step (anticipation). Eventually, only vehicles that did not brake at the previous time step are allowed to change lane.

We have chosen to model the on-ramp through a minimal perturbation approach : the on-ramp is simulated as a 200 meters long merging region on the right lane, on which we try to insert cars with a given rate. Insertion is performed in the middle of the largest gap available in the merging region - if it is large enough -, while the velocity of the new car is as large as security criteria allow it. If insertion is not possible, a new trial will be performed at the next time-step, until insertion is completed.

If all cars are identical, i.e. if they have the same preferred velocity, we find that the relaxation length can be very long. Fig. 11 gives such an example, where the relaxation lengths extend over tens of kilometers. This is obviously unphysical. At least two mechanisms could cause this slow relaxation. First, for the lane changing rules considered here, a lane change is triggered if the time headway between two vehicles becomes less than 1s. If vehicles move with the same speed, i.e. if they do not catch up, this criterium is not triggered and vehicles do not even try to pass each other. Second, if several vehicles are almost at the same position on neighboring lanes, they prevent each other from changing lane. These blocking structures may be quite persistent when vehicles move with the same average speed, though using asymmetric changing rules limits this effect (Knospe et al., 1999; Chowdhury et al., 1997).

When two types of vehicles, with different preferred velocities, are considered (Fig. 12), two regimes can be distinguished, as this is visible on the zoomed figure 13 of the right lane. A first rapid relaxation occurs on the first 400 meters. Then a much slower relaxation occurs, with a characteristic length of the order of 3 km - a value which is still much larger than the experimental ones. One explanation could be that in the first stage, the fastest vehicles rearrange themselves among the lanes; it takes much more time for the slow vehicles to reach a new equilibrium between the lanes, for the same reasons as those given above for the one-speed case.

We have compared the relaxation lengths measured on four different simulations, respectively with 1, 2, 4, or 8 types of vehicles (see figure 14). Each type of vehicle differs from the other ones through its maximal velocity. The set of maximal velocities for each simulation was chosen so that the mean (113 km/h) and the root mean square (10.8 km/h) of the velocities were the

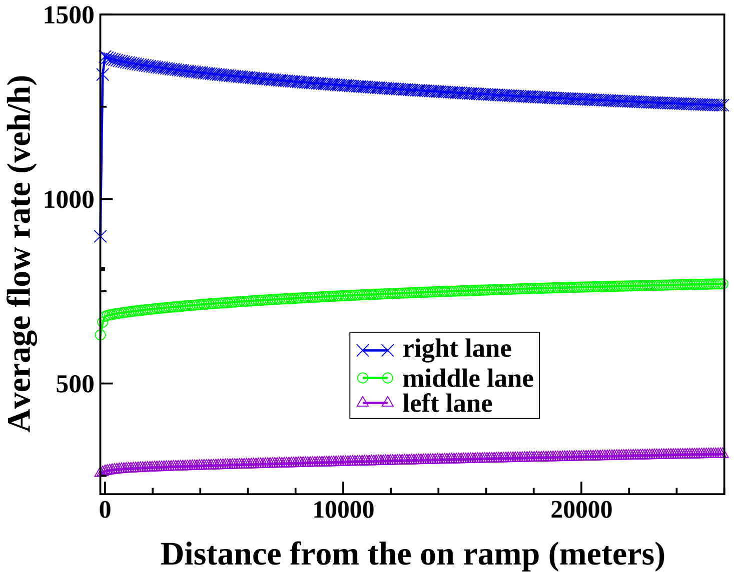

Figure 11: Flow rates on the right, middle and left lane (resp. blue, green and purple lines), as a function of the distance from the on-ramp, in the free flow regime. Only one type of vehicle is considered. The on-ramp flow rate represents 23% of the downwards total flow rate.

same for the four simulations (except that the root mean square is obviously zero when there is only one type of vehicles). The percentage of the total downstream flow rate coming from the on-ramp was kept constant ($\sim 20\%$). For more than one type of vehicles, as mentioned before, two regimes can be distinguished for the relaxation. Thus we were able to measure actually two lengths, one for the fast relaxation and another one for the slow relaxation. Both relaxation lengths converge when the number of vehicle types increases.

On the whole, relaxation lengths are reduced to realistic values when more types of vehicles are introduced, as this incites more often the vehicles to change lane and makes the persistence of local blocking configurations less likely. This underlines the importance of mixing vehicle types in order to get relaxation on realistic lengths - an feature that had not been pointed out by previous studies of the model.

We observed that with three types of cars, the relaxations lengths are not very sensitive to the relative densities of each type of cars - as long as the density of each type is not vanishing. These relative densities will rather determine the fractions of vehicles driving on each lane.

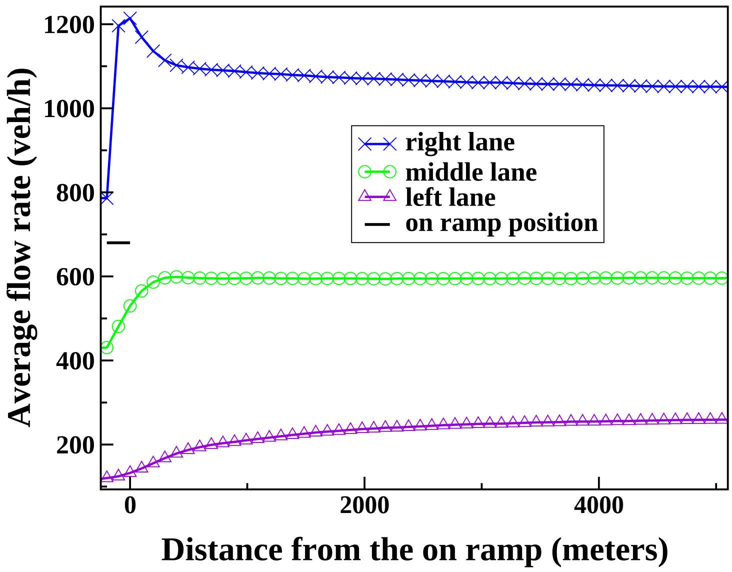

Figure 12: Flow rates on the right, middle and left lane (resp. blue, green and purple lines), as a function of the distance from the on-ramp, in the free flow regime. The thick black line indicates the location of the on-ramp. Two types of vehicles are considered, respectively with preferred velocities 108 and 130 km/h.

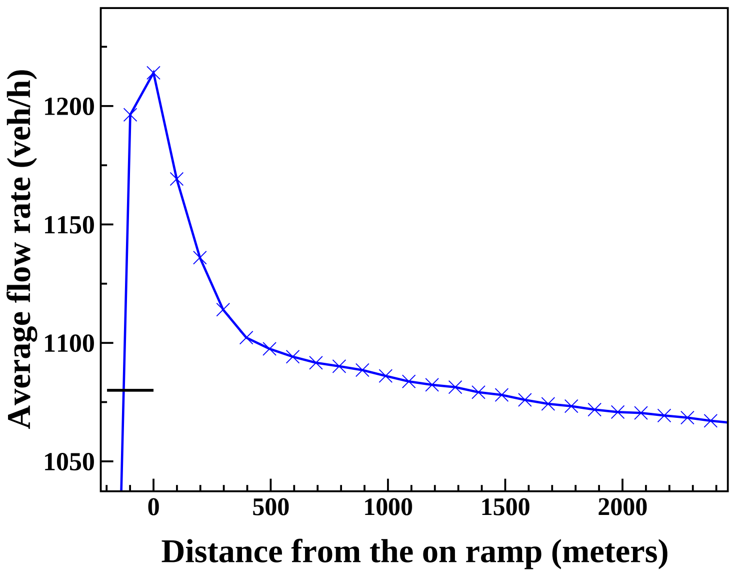

Figure 13: Zoom of the previous figure on the right lane flow. The thick black line indicates the location of the on-ramp.

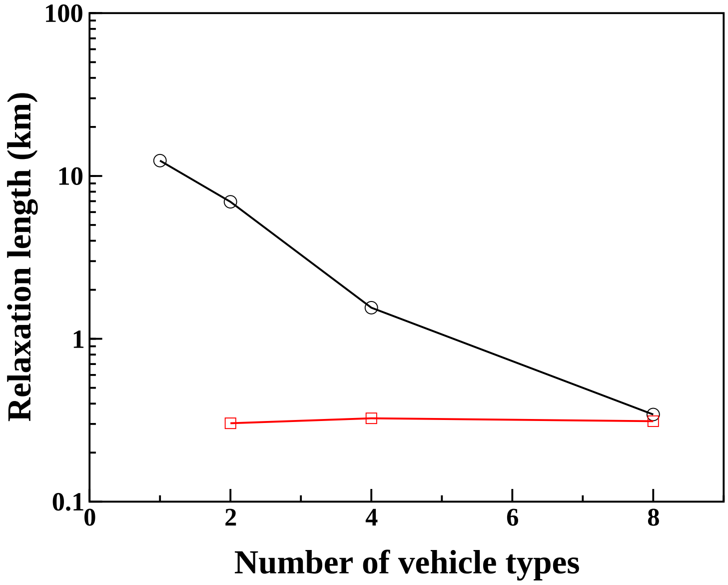

Figure 14: Relaxation lengths (km) on the right lane, respectively for the fast (squares) and slow (circles) relaxation processes, as a function of the number of vehicle types included in the simulations.

We did some simulations with the on-ramp switched off in order to determine the fundamental diagram of the main flow. We found that congested traffic occurs here for occupancies greater than about 10%.

Then the on-ramp is switched on. As we have seen, in the free flow regime, realistic relaxation lengths can be recovered provided enough vehicle types are mixed. We were not able to simulate the whole range of total flows observed for example in figure 6, as the capacity drop turned out to be too important once the on-ramp is switched on. Thus the remaining of our study will rather focus on the congested state. From now on, all the results will be obtained with four types of vehicles, having different preferred velocities (respectively 97, 108, 119, and 130 km/h), and inserted with equal probability.

If the on-ramp perturbation is applied to a highly congested state, an even more severe congestion is created at the level of the on-ramp on the right lane. In the case illustrated in figures 15 and 16, vehicles almost come to a stop when they arrive at the on-ramp level (the velocity goes down to 3 km/h). Immediately after the on-ramp, the vehicles going out from this extremely congested region accelerate and the occupancy decreases. The

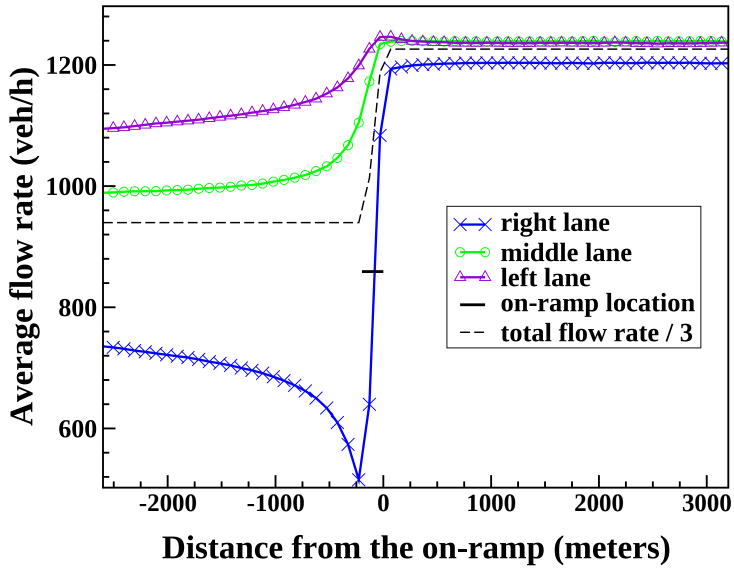

Figure 15: Flow rates on the right, middle and left lane (resp. blue, green and purple lines), as a function of the distance from the on-ramp, in a highly congested regime. The thick black line indicates the location of the on-ramp. The dashed line gives the total flow - which is of course conserved in the stationary state, except at the on-ramp level. The on-ramp flow rate is 860 veh/h.

strong congestion induced by the on-ramp has the effect that the relaxation slopes after the on-ramp are of opposite sign compared to the A6/A10 data : flow rate slowly increases on the right lane, and decreases on the left lane. Indeed, the vehicles that had moved to the left lanes to avoid the strong congestion at the level of the on-ramp partially switch back to the right once they have passed the obstacle. In Figs. 15 and 16, the on-ramp flow rate was quite high (860 veh/h). But the same behavior is observed also for weaker on-ramp flow rates (600 or 350 veh/h).

The severe congestion that occurs on the right lane, and the related inverted relaxation of lanes, was never observed on the A6/A10 data in spite of the very large flow rate from the on-ramp.

This unrealistic behavior of the model is a priori not due to the way vehicles from the on-ramp are inserted, as they are inserted in a way that should minimize the perturbation. It should rather be attributed to an overreaction of upstream vehicles. Indeed, vehicles in the simulation do not accept the transient short gaps imposed by the incoming vehicles, as a real driver would do.

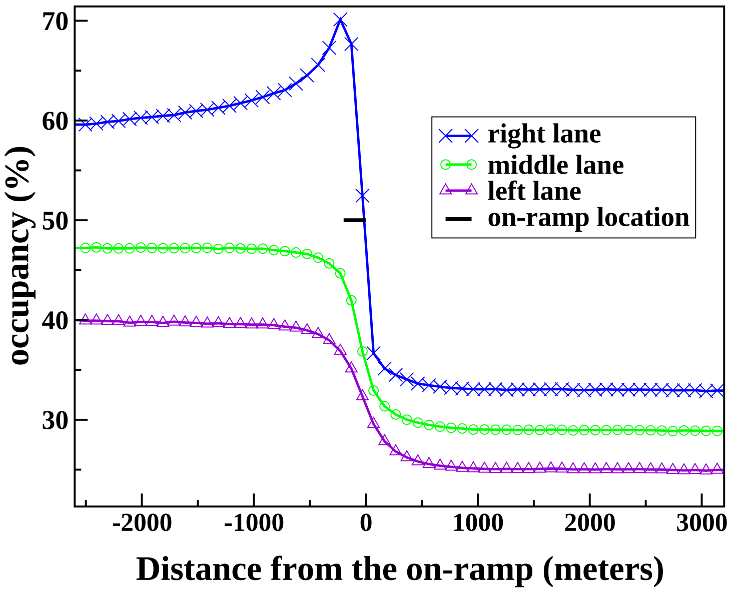

Figure 16: Occupancies on the right, middle and left lane (resp. blue, green and purple lines), as a function of the distance from the on-ramp, in a highly congested regime. The thick black line indicates the location of the on-ramp. Same simulation as in Fig. 15.

If the perturbation is applied to a less congested state, the lane flow profiles become qualitatively similar to those of real data, i.e. the slopes have the same signs both in simulations and empirical observations. However, the relaxation lengths are significantly longer than the experimental values found in Fig. 10. For example, for an upstream total flow rate equal to 3483 veh/h with upstream occupancies ranging from 30 to 42% on the various lanes, and for an on-ramp representing 10% of this upstream flow rate, the relaxation length measured on the right lane is equal to 640 meters, while the downstream occupancy is of the order of 25%.

One explanation for these long relaxation lengths could be that in the real world, vehicles who intend to change lane adjust their velocity so as to match the target lane velocity. Thus they can insert safely in quite small gaps. In cellular automata, no such anticipation occurs. As a consequence, vehicles do not change lane enough when there is a velocity difference between the lanes (here around 10km/h) and a large density.

If the incoming flow is even less congested (the upstream occupancy ranges from 25 to 31% on the three lanes), and with an on-ramp flow rate which still represents 10% of the upstream flow rate (3700 veh/h), we ob-

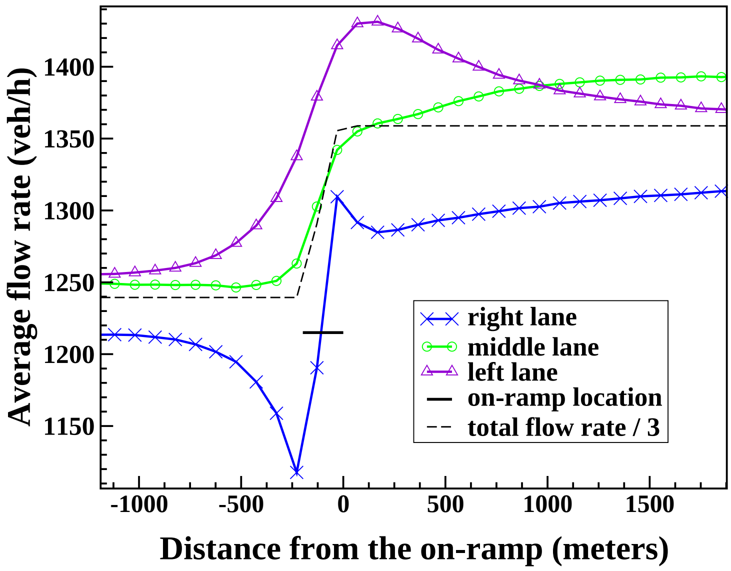

Figure 17: Flow rates on the right, middle and left lane (resp. blue, green and purple lines), as a function of the distance from the on-ramp. The thick black line indicates the location of the on-ramp. The dashed line gives the total flow rate - which is of course conserved in the stationary state, except at the on-ramp level. The on-ramp flow rate was 360 veh/h.

serve again inverted relaxation slopes. However, on the first lane, the flow rate after the on-ramp is not a monotonous function, as illustrated in Fig. 17. Actually two stages can be distinguished. In a first stage, which extends on the first kilometer, the occupancy decreases very rapidly (with a measured relaxation length of 190 meters). The velocity increases quite rapidly at the same time. At the beginning of this stage, the flow rate on the right lane decreases. This behavior could be related to one of the features of the model, the so-called "slow to start" rule, or VDR ("Velocity Dependent Randomization") rule (Barlović et al., 2002; Maerivoet and De Moor, 2004), according to which a vehicle with a small velocity has a weaker acceleration - a rule that is important to have metastability in the phase diagram and models a reaction time (Barlović et al., 2002; Appert and Santen, 2001).

In a second stage, a very slow relaxation occurs, with an increase of velocity and a decrease of occupancy. As the left lane has switched to a (high flow) free flow regime, flow rate decreases on this lane, while it increases on the two rightmost lanes which are still in the congested state. The relaxation length on which this occurs is at least of the order of 10 kilometers (the size of our system does not allow us to exclude that it could be even larger)!

In this stage, the velocities on the different lanes are quite different (20 km/h difference from one lane to the next one). As explained above, the lack of anticipation in the model makes it difficult for the vehicles to change lane when large velocity differences are observed. This can explain that the system relaxes over unrealistically large distances.

5. Conclusion

In this paper, we propose a procedure to test lane changing rules of microscopic models through the macroscopic response to a local perturbation. Indeed on-ramps create regions with a large demand of lane changes. Instead of studying the behavior of drivers at the microscopic scale - behavior which may vary from one person to another -, we study the response of the system at a macroscopic level, as an integration of all lane-changes and lateral interactions.

The approach is two-fold. First we need a better knowledge of the response of the highway flow to a local perturbation induced by an on-ramp. In this paper, we have presented an experimental study in which we have measured in particular the relaxation length for the flow rate on the right lane downwards the on-ramp. Though the distance between the measuring devices did not allow for very precise measurements, we found that all the relaxation lengths that we measured were of the order of hundreds of meters. In the free flow regime, these relaxation lengths were found to increase with the occupancy when the on-ramp represents a small fraction of the total downstream flow (typically 14%), while they become independent of the occupancy when the on-ramp contribution is large (typically 40%). The opposite phenomenon was observed in the congested regime, i.e. relaxation lengths decrease with the occupancy. It would be interesting to have measurements in other sites in order to test the generality of these observations.

Second, the experimental results must be compared to the simulations, in order to test how realistic the response is in the numerical model. To show how sensitive is this test, we have applied it to a cellular automata model, that had been quite well validated both for longitudinal interactions, and for reproducing realistic lane fractions on homogeneous highway stretches. It was known, however, that the lane changing rules of this model had deficiencies, but it had never been clearly pointed out. Here we propose a much more sensitive test, that immediately shows that the multi-lane behavior of the model is not realistic at the macroscopic level.

We have found in particular that in the free flow regime, it was very important to have a mixture of several types of vehicles, with different preferred velocities, in order to obtain realistic relaxation lengths.

In the congested regime, the response of the simulated system allowed to evidence some strong limitations of the model. Two features should in particular be responsible for the unrealistic behavior of the model. First, the lack of anticipation to adjust the velocity to the target lane before a lane change makes it difficult for the vehicles to change lane in dense traffic - or if they do, the perturbation induced on the target lane by the velocity difference is systematically overestimated.

Second, in real systems, the lane changing vehicles also accept temporarily shorter time headways, which relax afterwards (Laval and Leclercq, 2007; Smith, 1985). This effect, which is not included in cellular automata models, could also play a role in the global dynamics of the system. Correcting these model limitations and testing whether it indeed improves its macroscopic response will be left for future work.

This paper illustrates how macroscopic data can be used as a test for microscopic modeling rules. We hope that it will trigger some interest for more experimental and numerical studies on this relaxation phenomenon. It would be interesting to compare different sets of real data, in particular with a higher density of detectors. This would allow to test the robustness of our observations and to refine measurements. Besides, from the numerical point of view, a systematic comparison of microscopic models (cellular automata, car following...) remains to be done. The methodology presented here can be seen as a benchmark that allows to compare models even if they are based on very different methods.

It would be interesting too to understand more directly the link between the relaxation phenomenon that we describe here, and the capacity drop phenomenon.

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