INTRODUCTION TO tmQCD

Orsay-Bielefeld Lectures on
"Lattice Simulations and Quantum Fileds"

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Generalities

- tmQCD is a relative newcomer in the family of lattice fermion regularizations

- it consists in modifying the standard Wilson fermion matrix by adding a mass term , which is "twisted" in chiral space

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Pauli matrix in SU(2) flavour space

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Pauli matrix in SU(2) flavour space

- there are several advantages in such a choice:
  - natural infrared cutoff enables a safer approach to the chiral limit (and keeps us safe from exceptional configurations in the quenched approximation)
  - in many cases the renormalization properties of WMEs are simplified
  - in most cases of interest observable quantities are improved "automatically" (i.e. without Symanzik counter-terms in the action and the operators)

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- there is a price to pay: flavour symmetry is lost and so are parity and time reversal (recovered in the continuum limit)
- this mini-course emphasizes first principles, illustrated by selected results; it is NOT a complete review of the tmQCD state of the art

Classical tmQCD

Alpha Collab., R. Frezzotti, P.A. Grassi, S. Sint and P.Weisz, JHEP08 (2001) 058

Classical tmQCD

- for simplicity start with two degenerate flavours $\bar{\psi} = (\, \bar{u} \quad \bar{d} \,)$
- the classical QCD theory with SU(2) flavour symmetry is:

$$\mathcal{L} = \bar{\psi} \, [\, \not{D} + m + i \, \mu \, \tau^3 \, \gamma_5 \,] \, \psi$$

- **apparently** this is not QCD! (parity breaking? isospin braking? extra mass term?)
- **but** this theory is form invariant under chiral transformations in 3rd isospin direction, combined with spurionic transformations of the two mass parameters
- to see this, define first an **invariant mass** and a **twist angle**:

$$M = \sqrt{m^2 + \mu^2} \qquad \tan(\omega) = \frac{\mu}{m}$$

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standard mass squared

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- redefine fermionic fields through chiral rotations [$I_3(\alpha)$ - rotations]:

$$\psi \rightarrow \psi' = \exp\left[i \frac{\alpha}{2} \gamma_5 \tau^3 \right] \psi \qquad \bar{\psi} \rightarrow \bar{\psi}' = \bar{\psi} \exp\left[i \frac{\alpha}{2} \gamma_5 \tau^3 \right]$$

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Transformation angle

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- redefine mass parameters through spurionic transformations are:

$$m \rightarrow m' = m\cos(\alpha) + \mu\sin(\alpha)$$
$$\mu \rightarrow \mu' = \mu\cos(\alpha) - m\sin(\alpha)$$

Classical tmQCD

- the form invariance of the theory is:

$$\mathcal{L} \;=\; \bar{\psi}\left[\not{D} \;+\; m \;+\; i\,\mu\,\tau^3\,\gamma_5\right]\psi \;=\; \bar{\psi}\left[\not{D} \;+\; M\;\exp[i\omega\tau^3\gamma_5]\right]\psi$$

$$\downarrow$$

$$\mathcal{L}' \;=\; \bar{\psi}'\left[\not{D} \;+\; m' \;+\; i\,\mu'\,\tau^3\,\gamma_5\right]\psi' \;=\; \bar{\psi}'\left[\not{D} \;+\; M\;\exp[i\omega'\tau^3\gamma_5]\right]\psi'$$

- with the **same** invariant mass and a **new** twist angle

$$M' = M \qquad \tan(\omega') = \frac{\mu'}{m'}$$

$$\omega' = \omega - \alpha$$

Classical tmQCD

- the form invariance of the theory is:

$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\,\mu\,\tau^3\gamma_5\right]\psi = \bar{\psi}\left[\not{D} + M\,\exp[i\omega\tau^3\gamma_5]\right]\psi$$

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- with the **same** invariant mass and a **new** twist angle

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- we have a family of theories, prametrised by their twist angle
- they are **equivalent**, as they are linked by field and mass redefinitions
- the quark mass is given by the invariant mass ***M***

Classical tmQCD

- the form invariance of the theory is:

$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\mu\tau^3\gamma_5\right]\psi = \bar{\psi}\left[\not{D} + M\exp[i\omega\tau^3\gamma_5]\right]\psi$$

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- with the **same** invariant mass and a **new** twist angle

$$M' = M \qquad \tan(\omega') = \frac{\mu'}{m'}$$

$$\omega' = \omega - \alpha$$

- with $I_3(\alpha = \omega)$ - rotations we obtain $\boldsymbol{\omega' = 0 \Leftrightarrow \mu' = 0}$ and $\boldsymbol{m' = M}$

- i.e. the special case of zero twist angle is **QCD !!**

Classical tmQCD

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- with the **same** invariant mass and a **new** twist angle

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$$\omega' = \omega - \alpha$$

- with $I_3(\alpha = \omega - \pi/2)$ - rotations we obtain $\boldsymbol{\omega' = \pi/2 \Leftrightarrow m' = 0}$ and $\boldsymbol{\mu' = M}$

- this special case of interest is known as **fully twisted QCD or maximally twisted QCD!!**

Classical tmQCD

- symmetries are lost only **apparently**, since at the classical level QCD $\leftrightarrow$ tmQCD

- parity breaking? isospin braking?

$$\mathcal{L} = \bar{\psi}\left[\not{D} + m + i\,\mu\,\tau^3\gamma_5\right]\psi = \bar{\psi}\left[\not{D} + M\ \exp[i\omega\tau^3\gamma_5]\right]\psi$$

- QCD is obtained from tmQCD (defined at fixed **ω**) with chiral transformations in 3rd isospin direction [$I_3(\omega)$-rotations], combined with spurionic transformations of the two mass parameters:

$$\tan(\omega) = \frac{\mu}{m}$$

$$\psi \to \psi' = \exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\psi \qquad \bar{\psi} \to \bar{\psi}' = \bar{\psi}\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]$$

$$m \to m' = M$$
$$\mu \to \mu' = 0$$

- the symmetry transformations of the fermion fields in the tmQCD formalism are obtained by performing the opposite $I_3(-\omega)$-rotations to the standard symmetry transformations of the fields in QCD

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- twisted parity is $\boldsymbol{P_\omega}$

$$\tan(\omega) = \frac{\mu}{m}$$

$$\begin{aligned}
x = (x^0, \mathbf{x}) &\quad\longrightarrow\quad x' = (x^0, -\mathbf{x}) \\
A_0(x) &\quad\longrightarrow\quad A_0(x') \\
A_k(x) &\quad\longrightarrow\quad -A_k(x') \\
\psi(x) &\quad\longrightarrow\quad \gamma_0 \exp\left[i\omega\gamma_5\tau^3\right]\psi(x') \\
\bar{\psi}(x) &\quad\longrightarrow\quad \bar{\psi}(x') \exp\left[i\omega\gamma_5\tau^3\right]\gamma_0
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- twisted time-reversal is similarly ***T*$_\omega$**

$$\tan(\omega) = \frac{\mu}{m}$$

$$\begin{aligned} x = (x^0, \mathbf{x}) &\quad\rightarrow\quad x' = (-x^0, \mathbf{x}) \\ A_0(x) &\quad\rightarrow\quad -A_0(x') \\ A_k(x) &\quad\rightarrow\quad A_k(x') \\ \psi(x) &\quad\rightarrow\quad i\,\gamma_0\gamma_5 \exp\left[i\omega\gamma_5\tau^3\right]\psi(x') \\ \bar{\psi}(x) &\quad\rightarrow\quad -i\,\bar{\psi}(x') \exp\left[i\omega\gamma_5\tau^3\right]\gamma_5\gamma_0 \end{aligned}$$

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$$\mathcal{L} = \bar{\psi} \left[\not{D} + m + i\,\mu\,\tau^3\,\gamma_5 \right] \psi = \bar{\psi} \left[\not{D} + M \,\exp[i\omega\tau^3\gamma_5] \right] \psi$$

- NB: instead of twisted parity $\boldsymbol{P_\omega}$ we may have standard parity $\boldsymbol{P_0}$ with twisted mass sign flip

$$\begin{array}{rcl} x = (x^0, \mathbf{x}) & \rightarrow & x' = (x^0, -\mathbf{x}) \\ A_0(x) & \rightarrow & A_0(x') \\ A_k(x) & \rightarrow & -A_k(x') \\ \psi(x) & \rightarrow & \gamma_0\ \psi(x') \\ \bar{\psi}(x) & \rightarrow & \bar{\psi}(x')\ \gamma_0 \\ \mu & \rightarrow & -\mu \end{array}$$

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- NB: instead of twisted parity $\boldsymbol{P_\omega}$ we may have standard parity $\boldsymbol{P_0}$, combined with (spurionic) twisted mass sign flip

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- similarly for time-reversal $\boldsymbol{T_0} \otimes$ **[μ → -μ]**

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- twisted vector symmetry (isospin) is ***SU$_V$(2)$_\omega$***

$$\tan(\omega) = \frac{\mu}{m}$$

$$\psi(x) \rightarrow \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[i\frac{\theta^a}{2}\tau^a\right]\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\psi(x)$$

$$\bar{\psi}(x) \rightarrow \bar{\psi}(x)\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[-i\frac{\theta^a}{2}\tau^a\right]\exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]$$

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- by analogy, twisted axial symmetry is ***$SU_A(2)_\omega$***

$$\tan(\omega) = \frac{\mu}{m}$$

$$\begin{aligned}
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\end{aligned}$$

- axial symmetry valid at $M = 0$

Classical tmQCD

- the $I_3(\omega)$-rotations relating QCD ↔ tmQCD give operator correspondences

$$
\begin{aligned}
\mathcal{V}_\mu^a &= \cos(\omega)\, V_\mu^a + \epsilon^{3ab} \sin(\omega)\, A_\mu^b \qquad a = 1,2 \\
\mathcal{A}_\mu^a &= \cos(\omega)\, A_\mu^a + \epsilon^{3ab} \sin(\omega)\, V_\mu^b \qquad a = 1,2 \\
\mathcal{V}_\mu^3 &= V_\mu^3 \\
\mathcal{A}_\mu^3 &= A_\mu^3
\end{aligned}
$$

$$Q_\Gamma^a = \bar{\psi}\, \Gamma \frac{\tau^a}{2}\, \psi$$

defined in QCD

defined in tmQCD

$$S^0 = \bar{\psi}\, \psi$$

$$\tan(\omega) = \frac{\mu}{m}$$

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\end{aligned}
$$

$$Q^a_\Gamma = \bar\psi\, \Gamma\, \frac{\tau^a}{2}\, \psi$$

defined in QCD

defined in tmQCD

$$S^0 = \bar\psi\, \psi$$

$$
\begin{aligned}
\mathcal{P}^a &= P^a \qquad && a = 1,2 \\
\mathcal{P}^3 &= \cos(\omega)\, P^3 + \frac{i}{2} \sin(\omega)\, S^0 \\
\mathcal{S}^0 &= \cos(\omega)\, S^0 + 2i \sin(\omega)\, P^3
\end{aligned}
$$

$$\tan(\omega) = \frac{\mu}{m}$$

Classical tmQCD

- similar correspondences occur in Ward identiites
- in tmQCD the PCVC is

$$\partial_\mu V_\mu^a \;=\; -2\mu\,\epsilon^{3ab}\,P^b$$

- in tmQCD the PCAC is

$$\partial_\mu A_\mu^a \;=\; 2m\,P^a \;+\; i\,\mu\,\delta^{3a}\,S^0$$

$$Q_\Gamma^a \;=\; \bar\psi\,\Gamma\,\frac{\tau^a}{2}\,\psi$$
$$S^0 \;=\; \bar\psi\;\psi$$

- in terms of the QCD currents and densities, they become the standard expressions

Lattice tmQCD

Alpha Collab., R. Frezzotti, P.A. Grassi, S. Sint and P. Weisz, JHEP08 (2001) 058

Lattice tmQCD

- QCD ↔ tmQCD equivalence carries over to the renormalized quantum level
- Ingredients:
  - chiral symmetry of Ginsparg-Wilson (GW) fermions
  - mass-independent renormalization scheme
  - universality of different lattice regularizations in the continuum limit
  - twist angle tuned to ratio of renormalized masses $\tan(\omega) \; = \; \mu_{\rm R} \, / \, m_{\rm R}$
- QCD ↔ tmQCD equivalence proceeds through linear mapping between renormalized Green functions
- regularize QCD and tmQCD with GW fermions
- GW chiral symmetry guarantees the same considerations of a trivial QCD ↔ tmQCD equivalence as in the classical case are valid (with minor caveats)
- example: **bare** Green function of the scalar operator (chiral condensate)

$$\Big[< \cdots \mathcal{S}^0 \cdots > \Big]_{\rm GW}^{\rm QCD} \; = \; \Big[\cos(\omega) \; < \cdots S^0 \cdots > \; + \, i \; \sin(\omega) \; < \cdots P^3 \cdots > \Big]_{\rm GW}^{\rm tmQCD}$$

Lattice tmQCD

- QCD ↔ tmQCD equivalence between bare Green functions with GW regularization

$$\Big[< \cdots \mathcal{S}^0 \cdots > \Big]^{\rm QCD}_{\rm GW} \; = \; \Big[\cos(\omega) \; < \cdots S^0 \cdots > \; + i \; \sin(\omega) \; < \cdots P^3 \cdots > \Big]^{\rm tmQCD}_{\rm GW}$$

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- QCD ↔ tmQCD equivalence between bare Green functions with GW regularization

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- QCD ↔ tmQCD equivalence carries over to renormalized quantities, due to mass independent renormalization schemes (i.e. S^0 and P^3 in both QCD and tmQCD have the same renormalization constant $Z_S = Z_P = Z$)

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Lattice tmQCD

- QCD $\leftrightarrow$ tmQCD equivalence between bare Green functions with GW regularization

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- since QCD $\leftrightarrow$ tmQCD equivalence holds between renormalized (continuum) Green functions (proved with GW regularization), evoking universality we claim that this is also true for renormalized (continuum) Green functions computed with any other lattice regularization; e.g. tmQCD wih Wilson fermions

$$\Big[< \cdots \mathcal{S}^0 \cdots >_{\rm R} \Big]^{\rm QCD} = \Big[\cos(\omega) < \cdots S^0 \cdots >_{\rm R} + i \sin(\omega) < \cdots P^3 \cdots >_{\rm R} \Big]^{\rm tmQCD}$$

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$$\Big[< \; \cdots \mathcal{S}^0 \cdots \; >_{\rm R} \; \Big]^{\rm QCD} \;=\; \Big[\cos(\omega) < \; \cdots S^0 \cdots \; >_{\rm R} \; + i \sin(\omega) < \; \cdots P^3 \cdots \; >_{\rm R} \; \Big]^{\rm tmQCD}$$

- QCD $\leftrightarrow$ tmQCD equivalence amounts to operator transcriptions in lattice tmQCD

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) \; + \; \frac{1}{2}ar\nabla^*_\mu\nabla_\mu \; + \; m_0 \; + \; i\mu_q\tau^3\gamma_5\right]\psi \\
&= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) \; + \; \frac{1}{2}ar\nabla^*_\mu\nabla_\mu \; + \; M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi
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&= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi
\end{aligned}
$$

naive derivative

Lattice tmQCD

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\end{aligned}
$$

Wilson term

- Wilson term cures the fermion doubling problem

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + \frac{1}{2}ar\nabla_\mu^*\nabla_\mu + m_0 + i\mu_q\tau^3\gamma_5\right]\psi \\ &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) + \frac{1}{2}ar\nabla_\mu^*\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi\end{aligned}$$

bare
standard
mass

- bare standard mass renormalizes as in standard Wilson fermions:

$$m_{\mathrm{R}} = Z_m\,[m_0 - m_{\mathrm{cr}}] = Z_{\mathrm{S}}^{-1}\,[m_0 - m_{\mathrm{cr}}]$$

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&= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi
\end{aligned}
$$

bare
twisted
mass

- bare twisted mass renormalizes multiplicatively (protected by Ward identities):

$$
\mu_{\mathrm{R}} = Z_\mu\,\mu_q = Z_{\mathrm{P}}^{-1}\,\mu_q
$$

Lattice tmQCD

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$$

- Wilson term causes loss of twisted symmetries: ***P*$_\omega$, *T*$_\omega$**

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\begin{aligned}\mathcal{L} &= \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi \\ &= \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi\end{aligned}$$

- Wilson term causes loss of twisted symmetries: ***P_ω*, *T_ω***
- **Parity** survives if combined either with flavour exchange (defined as ***P_F^1*** , ***P_F^2***) ...

$$\psi(x)\rightarrow i\,\gamma_0\,\tau^1\,\psi(x') \qquad \bar\psi(x)\rightarrow -i\,\bar\psi(x')\,\gamma_0\,\tau^1$$

$$\psi(x)\rightarrow i\,\gamma_0\,\tau^2\,\psi(x') \qquad \bar\psi(x)\rightarrow -i\,\bar\psi(x')\,\gamma_0\,\tau^2$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\mathcal{L} = \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi$$

$$= \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi$$

- Wilson term causes loss of twisted symmetries: ***P*$_\omega$, *T*$_\omega$**
- ... or with a sigh flip of the twisted mass (defined as ***P*** ⊗[μ → - μ])

$$\psi(x) \to i\,\gamma_0\,\psi(x') \qquad \bar\psi(x) \to -i\,\bar\psi(x')\,\gamma_0$$

$$\mu_q \quad \to \quad -\,\mu_q$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

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$$\mu_q \quad\to\quad -\,\mu_q$$

- The same holds for time reversal: ***T***$_F^1$, ***T***$_F^2$, ***T*** ⊗[μ → - μ] are symmetries

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi \\ &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi\end{aligned}$$

- Wilson term causes loss of twisted vector symmetry: ***$SU_V(2)_\omega$***; i.e. flavour symmetry is **hard**-broken (Wilson term) in tmQCD
- However it is not completely broken; the subgroup ***$U_V^3(1)$*** survives (NB: ω-dependence drops out!)

$$\begin{aligned}\psi(x) &\rightarrow \exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[i\frac{\theta^a}{2}\tau^a\right]\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\psi(x) \\ \bar{\psi}(x) &\rightarrow \bar{\psi}(x)\exp\left[i\frac{\omega}{2}\gamma_5\tau^3\right]\exp\left[-i\frac{\theta^a}{2}\tau^a\right]\exp\left[-i\frac{\omega}{2}\gamma_5\tau^3\right]\end{aligned}$$

Lattice tmQCD

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$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi \\
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- However it is not completely broken; the subgroup ***$U_V^3(1)$*** survives (NB: ω-dependence drops out!)

$$
\begin{aligned}
\psi &\rightarrow \exp\left[i\frac{\alpha^3}{2}\tau^3\right]\psi \\
\bar{\psi} &\rightarrow \bar{\psi}\,\exp\left[-i\frac{\alpha^3}{2}\tau^3\right]
\end{aligned}
$$

Lattice tmQCD

- lattice tmQCD definition: Wilson fermions + twisted mass term:

$$\begin{aligned}\mathcal{L} &= \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla^*_\mu) + \boxed{\frac{1}{2}ar\nabla^*_\mu\nabla_\mu} + m_0 + i\mu_q\tau^3\gamma_5\right]\psi \\ &= \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + M_0\exp[i\omega_0\tau^3\gamma_5]\right]\psi\end{aligned}$$

- this hard ***$SU_V(2)_\omega$ → $U_V^3(1)$*** breaking causes a lack of degeneracy between the neutral pion π^0 and the two charged pions $\pi^\pm$
- It is a discretization effect which vanishes in the continuum limit (***$SU_V(2)$*** restoration)

$$\begin{aligned}\psi &\rightarrow \exp\left[i\frac{\alpha^3}{2}\tau^3\right]\psi \\ \bar\psi &\rightarrow \bar\psi\,\exp\left[-i\frac{\alpha^3}{2}\tau^3\right]\end{aligned}$$

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\end{aligned}
$$

- axial symmetry ***$SU_A(2)_\omega$***, broken **softly** by mass term M_0, also **hard**-broken by Wilson term in standard fashion
- also this symmetry is restored in the continuum and chiral limits

1st advantage: IR zero-mode regularization

Alpha Collab., R. Frezzotti, P.A. Grassi, S. Sint and P. Weisz, JHEP08 (2001) 058

IR zero-mode regularization

- Wilson fermion matrix $M = D_W + m_q$ has spurious zero-modes at small quark mass (lattice artefacts)
- In quenched simulations they cause exceptional configurations which impede simulations at masses lower than, say, half the strange quark mass; $m_q \leq m_s / 2$
- In un-quenched simulations the fermion determinant suppresses these zero-modes in MC, but an IR cutoff could still be helpful to the approach of the chiral limit
- tmQCD introduces an IR mass cutoff (the twisted mass!) which facilitates the approach to small mass regime

$$\bar{\psi}\ M_W\ \psi \ = \ (\bar{u}\ \ \bar{d}) \begin{pmatrix} D_W + m_0 + i\mu\gamma_5 & 0 \\ 0 & D_W + m_0 - i\mu\gamma_5 \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}$$

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$$\begin{aligned} \det M_W &= \det \begin{pmatrix} D_W + m_0 + i\mu\gamma_5 & 0 \\ 0 & D_W + m_0 - i\mu\gamma_5 \end{pmatrix} \\ &= \det \left[(D_W + m_0)^\dagger (D_W + m_0) + \mu^2 \right] \end{aligned}$$

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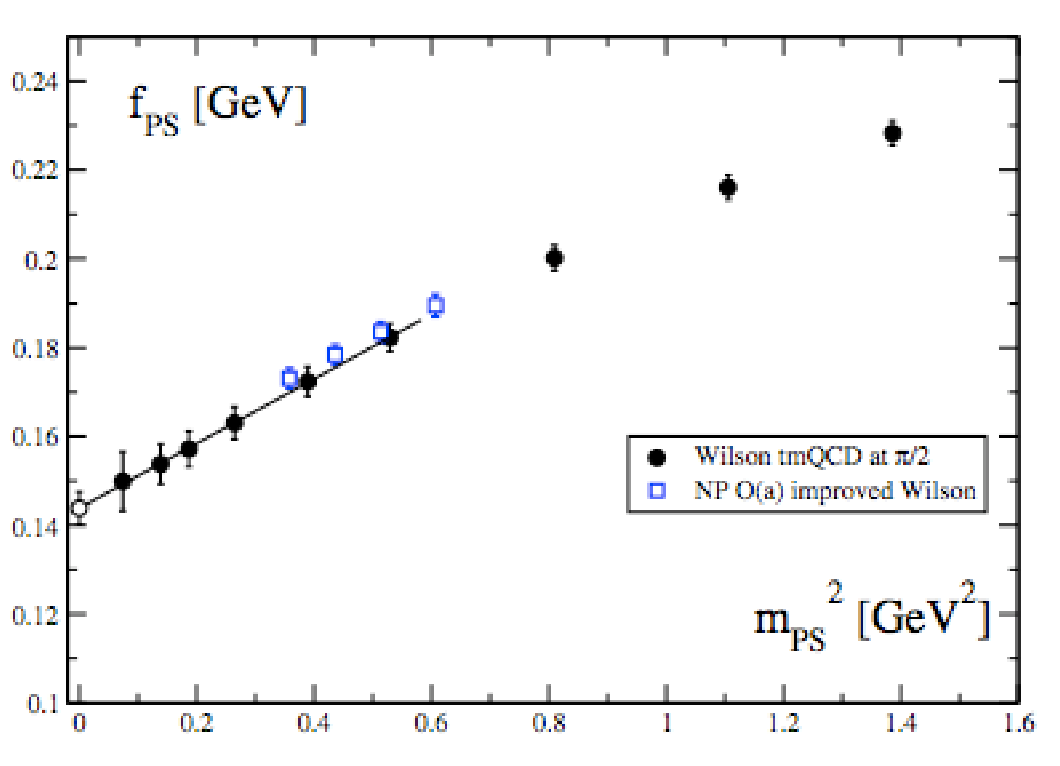

χLF Collab. K. Jansen, M. Papiutto, A. Shindler, C. Urbach, I. Wetzorke JHEP09 (2005) 071

Alpha Collab. M.Guagnelli, J.Heitger, R.Sommer, H.Wittig Nucl. Phys. B560 (1999) 465

2nd advantage: simplified renormalization

Alpha Collab., R. Frezzotti, P.A. Grassi, S. Sint and P.Weisz, JHEP08 (2001) 058

tmQCD and renormalization

- Wilson fermions renormalization may be complicated due to loss of chiral symmetry
- this may be simplified in tmQCD through judicious choices of the twist angle
- example 1: the chiral condensate $< S^0 > = < \bar{\psi}\psi >$
  - multiplicatively renormalizable with chirally symmetric regularization

$$< \bar{\psi}\psi >_R = Z_{s^0} < \bar{\psi}\psi >$$

Z_S is log.ly divergent

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$$< \bar{\psi}\psi >_R = Z_{s^0} \Big[< \bar{\psi}\psi > + \frac{C(g_0^2)}{a^3} \Big]$$

- without chiral symmetry, the condensate mixes with the identity operator

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- power divergences are vigorous and would better be avoided
- other operators are mult.ly renormalizable: $P_R^3 = [\bar{\psi}\tau^3\gamma_5\psi]_R = Z_P P^3$
- the same renormalization constants Z_S (Z_P) apply to S^0 (P^3) in both QCD and tmQCD (in mass-independent renormalization schemes)

tmQCD and renormalization

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- example 1: the chiral condensate $< S^0 > = < \bar{\psi}\psi >$
  - multiplicatively renormalizable with chirally symmetric regularization
  - additive renormalization (power subtraction) with Wilson fermions
- recall that the renormalized condensate insertion is computed from the bare tmQCD theory as:

$$\Big[< \cdots \mathcal{S}^0 \cdots >_{\mathrm{R}} \Big]^{\mathrm{QCD}} = \Big[\cos(\omega) < \cdots S^0 \cdots >_{\mathrm{R}} + i \sin(\omega) < \cdots P^3 \cdots >_{\mathrm{R}} \Big]^{\mathrm{tmQCD}}$$

- for twist angle $\omega = \pi/2$ this means that in tmQCD the condensate is obtained from the mult.ly renormalizable pseudoscalar density

$$< \bar{\psi}\psi > = i Z_P \left[< P^3 > \right]^{\mathrm{tmQCD}}$$

tmQCD and renormalization

- Wilson fermions renormalization may be complicated due to loss of chiral symmetry
- this may be simplified in tmQCD through judicious choices of the twist angle
- example II: the pion/kaon decay constant
  - with Wilson fermions, axial current normalization factor Z_A required
  - using tmQCD it is obtained without any (re)normalization factor
- in maximally tmQCD symmetries appear lopsided; this is also true of Ward identities
- PCVC is exact:
  - $\partial_\mu^* < \tilde{V}_\mu^1(x)\ P^2(0) > = \ -2\mu < P^2(x)\ P^2(0) >$

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lattice backward derivative

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lattice backward derivative

point-split vector current

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$$m_\pi < 0\,|\,\tilde{V}_0^1(0)\,|\,\pi > = 2\mu < 0\,|\,P^2(0)\,|\,\pi >$$

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$$m_\pi\ < \ 0\,|\,\tilde{V}^1_0(0)\,|\,\pi > = \ 2\mu < \ 0\,|\,P^2(0)\,|\,\pi >$$

- this fully tmQCD bare vector current in the continuum gives the axial current
  - $$\left[< \ 0\,|A^1_\mu(0)\,|\,\pi >_R \right]^{\mathrm{QCD}} \ = \ \lim_{a\to 0} \left[< \ 0\,|\,\tilde{V}^1_\mu(0)\,|\,\pi > \right]^{\mathrm{tmQCD}}$$

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- this may be simplified in tmQCD through judicious choices of the twist angle
- example II: the pion/kaon decay constant
  - with Wilson fermions, axial current normalization factor Z_A required
  - using tmQCD it is obtained without any (re)normalization factor
- in maximally tmQCD symmetries appear lopsided; this is also true of Ward identities
- PCVC is exact:
  - $$\partial_\mu^* < \tilde{V}_\mu^1(x)\ P^2(0) > = -2\mu < P^2(x)\ P^2(0) >$$

$$m_\pi < 0\,|\,\tilde{V}_0^1(0)\,|\,\pi > = 2\mu < 0\,|\,P^2(0)\,|\,\pi >$$

- this fully tmQCD bare vector current in the continuum gives the axial current
  - $$\left[< 0\,|A_\mu^1(0)\,|\,\pi >_R \right]^{\mathrm{QCD}} = \lim_{a\to 0} \left[< 0\,|\,\tilde{V}_\mu^1(0)\,|\,\pi > \right]^{\mathrm{tmQCD}}$$
- thus the pion decay constant is

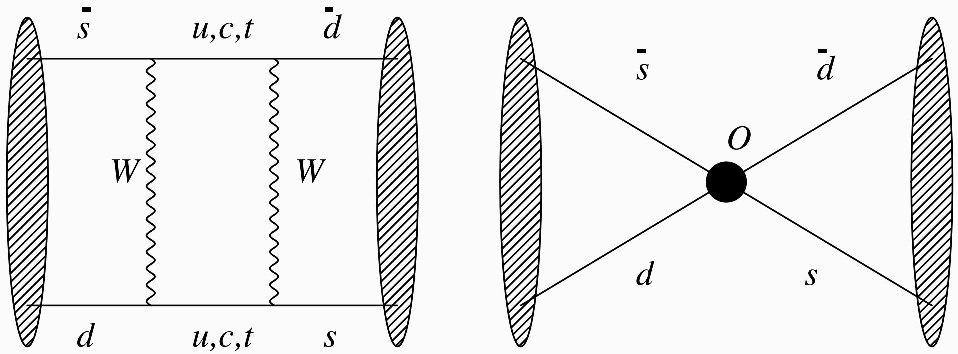
$$f_\pi = \frac{[< 0\,|\,A_0^1(0)\,|\,\pi >_R]^{\mathrm{QCD}}}{m_\pi} = \lim_{a\to 0} \frac{2\mu}{m_\pi^2} < 0\,|\,P^2(0)\,|\,\pi >$$

tmQCD and renormalization

- Wilson fermions renormalization may be complicated due to loss of chiral symmetry
- this may be simplified in tmQCD through judicious choices of the twist angle
- example III: the kaon B-parameter
    - indirect CP-violation
    - can be expressed in terms of K^0 - K^0 mixing
    - dominant EW process is FCNC (2 W exchange)

$$|\epsilon_K| \approx C_\epsilon\, \hat{B}_K\, \mathrm{Im}\{V_{td}^* V_{ts}\}\, \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2 S_0(x_t)]\}$$

long distance NP



$$\boxed{\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2\, m_K^2}}$$

$$\hat{O}^{\Delta S=2} = [\bar{s}\gamma_\mu^L d]\ [\bar{s}\gamma_\mu^L d]$$

B_K – a renormalisation classic

In the presence of explicit chiral symmetry breaking four-fermion operators of different chiralities mix under renormalisation.

Martinelli 1984; Bernard, Draper, (Hockney), Soni 1987, 1998; Gupta et al. 1993; Donini et al. 1999

$$O^{\Delta S=2} = [\underbrace{(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)}_{O_{\mathrm{VV+AA}}}] - [\underbrace{(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu\gamma_5 d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu d)}_{O_{\mathrm{VA+AV}}}]$$

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$$\bar{O}_{\mathrm{VV+AA}} = \lim_{a\to 0} Z_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O_{\mathrm{VV+AA}}(a) + \sum_{k=1}^{4} \Delta_k(g_0^2) O_k(a)\right]$$

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Vanishes if chiral symmetry is preserved (at least <u>partially</u>)

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Vanishes if chiral symmetry is preserved (at least partially)

Vanishes for staggered, GW, DW fermions

B_K – a renormalisation classic

In the presence of explicit chiral symmetry breaking four-fermion operators of different chiralities mix under renormalisation.

Martinelli 1984; Bernard, Draper, (Hockney), Soni 1987,1998; Gupta et al. 1993; Donini et al. 1999

$$O^{\Delta S=2} = \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu\gamma_5 d)]}_{O_{\mathrm{VV+AA}}} - \underbrace{[(\bar{s}\gamma_\mu d)(\bar{s}\gamma_\mu\gamma_5 d) + (\bar{s}\gamma_\mu\gamma_5 d)(\bar{s}\gamma_\mu d)]}_{O_{\mathrm{VA+AV}}}$$

$$\bar{O}_{\mathrm{VV+AA}} = \lim_{a\to 0} Z_{\mathrm{VV+AA}}(g_0^2, a\mu)\left[O_{\mathrm{VV+AA}}(a) + \sum_{k=1}^{4}\Delta_k(g_0^2)O_k(a)\right]$$

$$\bar{O}_{\mathrm{VA+AV}} = \lim_{a\to 0} Z_{\mathrm{VA+AV}}(g_0^2, a\mu)\, O_{\mathrm{VA+AV}}(a)$$

Protected from mixing by discrete symmetries *C P S(s ↔ d)*

Getting rid of mixing

- Straightforward option: preserve chiral symmetry — possibly exactly.

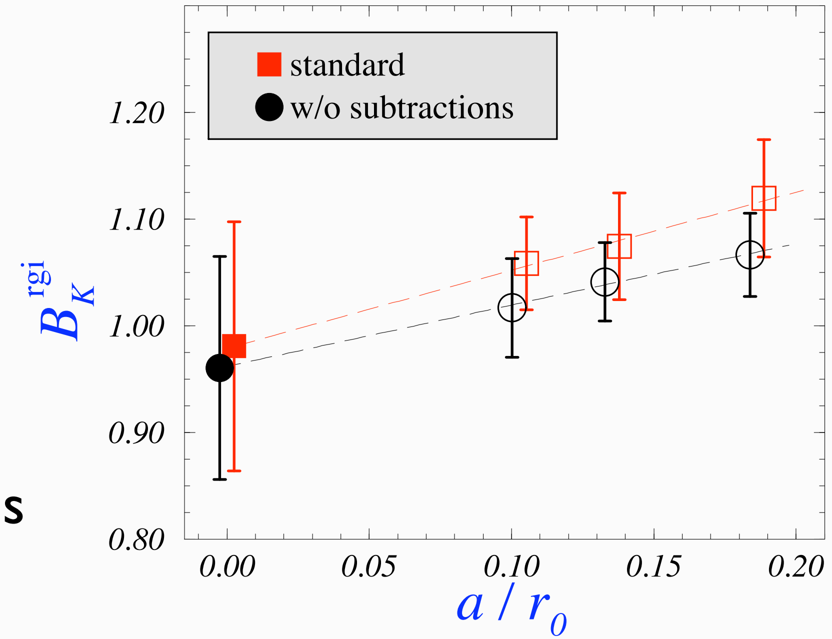
- Wilson I: axial Ward identity (3-point function with O_{VV+AA} → 4-point function with O_{VA+AV})

D.Becirevic et al. Phys.Lett.B487(2000)74; Eur.Phys.J.C37(2004)315

$$< \bar{K}^0 |\ \delta O_R\ | K^0 > \quad = \quad < \bar{K}^0 |\ O_R\ [\partial_\mu A_\mu - 2mP]\ | K^0 >$$

$$\delta O_R \quad = \quad [O_{VV+AA}]_R$$

$$O_R \quad = \quad [O_{VA+AV}]_R$$



  - subtractions traded off for fluctuations

Getting rid of mixing

- Straightforward option: preserve chiral symmetry — possibly exactly.

- Wilson 1: axial Ward identity (3-point function with O_{VV+AA} → 4-point function with O_{VA+AV})

D.Becirevic et al. Phys.Lett.B487(2000)74; Eur.Phys.J.C37(2004)315

- Wilson 2: tmQCD (3-point function with O_{VA+AV})

ALPHA M.Guagnelli, J.Heitger, C.Pena, S.Sint, A.V. JHEP 03 (2006) 088

F.Palombi, C.Pena, S.Sint JHEP 03 (2006) 089

ALPHA P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 749 (2006) 69

ALPHA P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 776 (2007) 258

- tmQCD bonus: push safely towards low quark masses in quenched simulations.

tmQCD regularisations for B_K

π/2 strategy:

$$S = \sum_{x,y} \{\bar\psi_\ell(x)\,[D_{\rm w,sw} + m_\ell + i\mu_\ell\gamma_5\tau_3]\,(x,y)\psi_\ell(y) \;+\; \bar s(x)\,[D_{\rm w,sw} + m_s]\,(x,y)s(y)\}$$

$m_\ell,\ \mu_\ell$ tuned to have $m_{\ell,\rm R} = 0$

π/4 strategy (specially devised for quenched case):

$$S = \sum_{x,y} \{\bar\psi(x)\,[D_{\rm w,sw} + m_{\hat 0} + i\mu_{\hat q}\gamma_5\tau_3]\,(x,y)\psi(y)\}$$

$m_{\hat 0},\ \mu_q$ tuned to have $m_{\rm R} = \mu_{\rm R}$

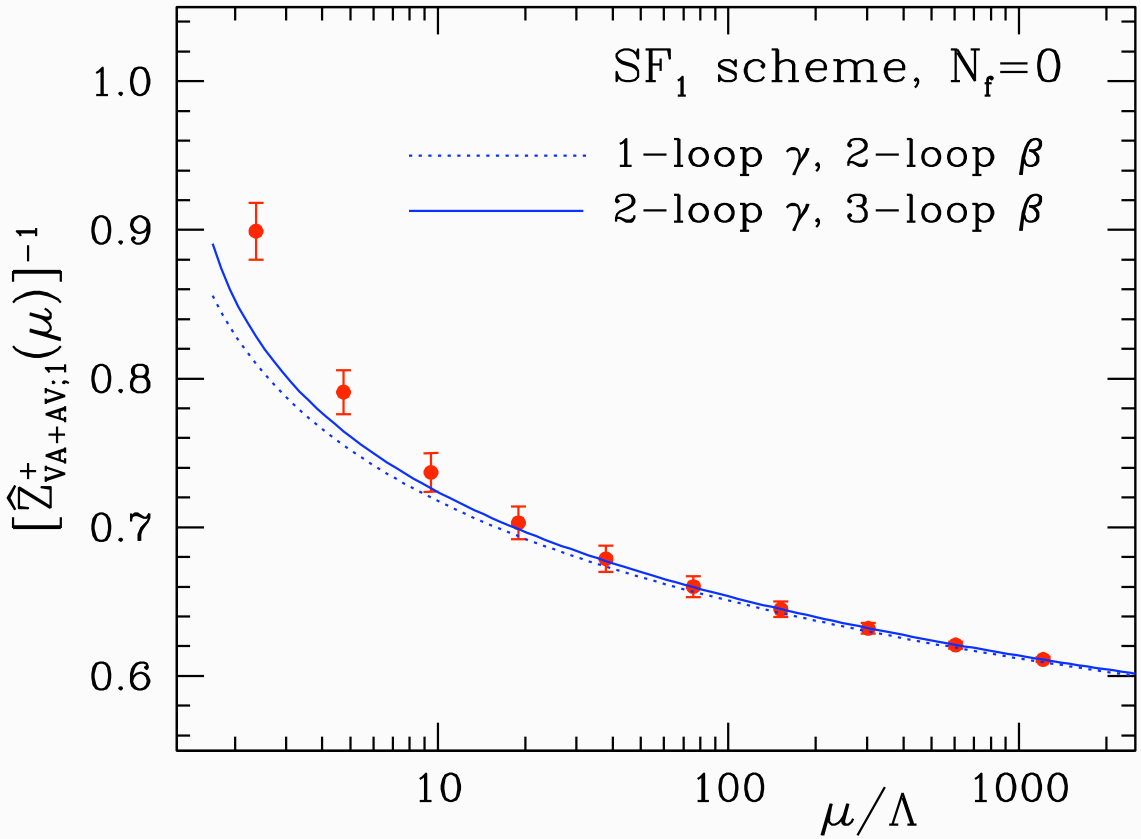
in both cases: $O_{\rm VV+AA} \xrightarrow{\text{twist}} O_{\rm VA+AV}$

NB: we never have *only* fully twisted quarks → Frezzotti-Rossi O(a) improvement argument does not apply.

Approach to continuum: non-perturbative renormalisation

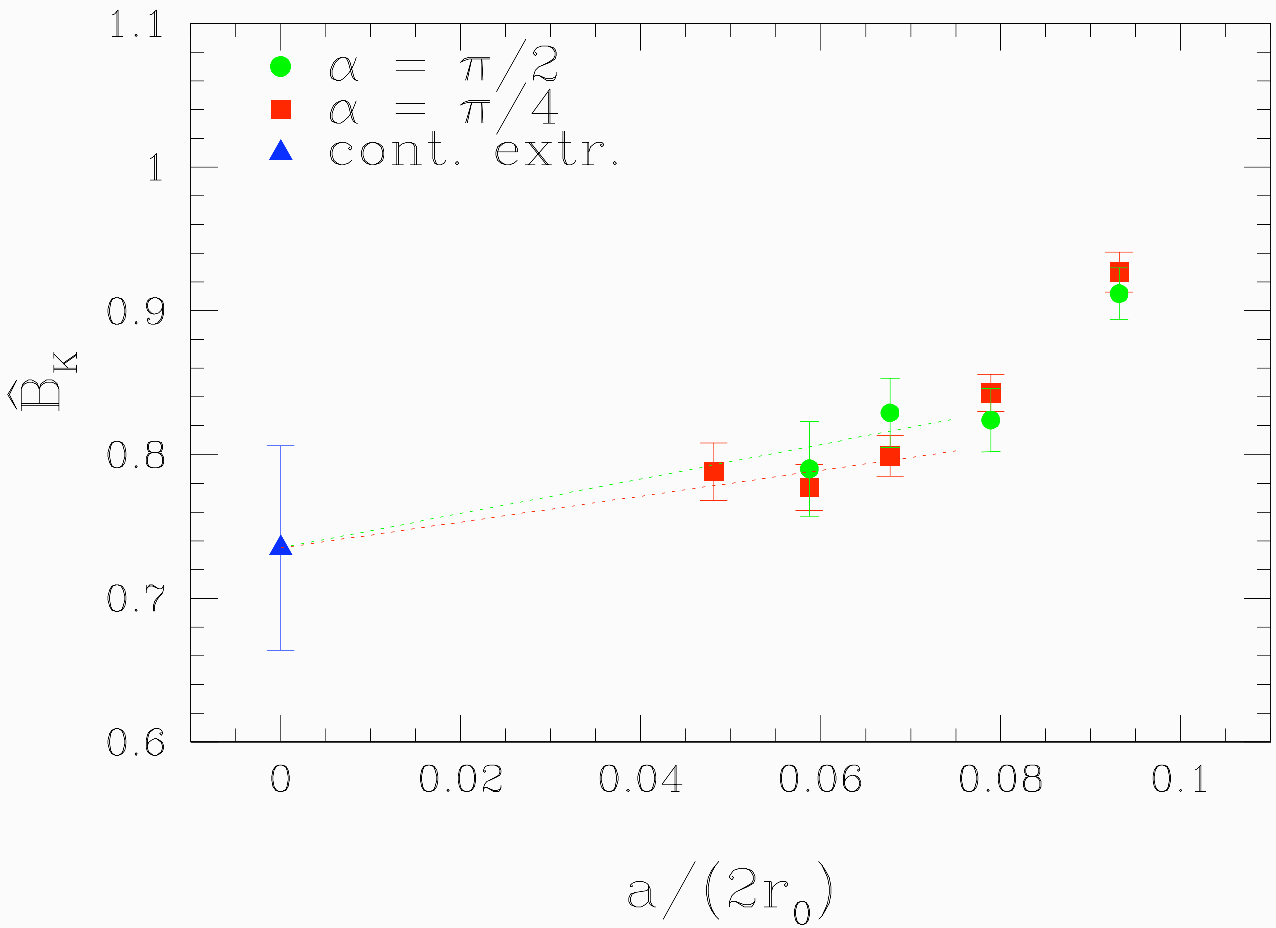
ALPHA, Guagnelli et al., JHEP 03 (2006) 088 ALPHA, Palombi et al., JHEP 03 (2006) 089

- SF technique via finite size scaling: split renormalisation into
  - Renormalisation at a low, hadronic scale where contact with typical large-volume values of β is made.
  - NP running to very high scales (~100 GeV) where contact with PT is made.



Continuum limit

- Combined linear extrapolation of the two regularisations, using ALPHA determination of current normalisations and improvement coefficients.

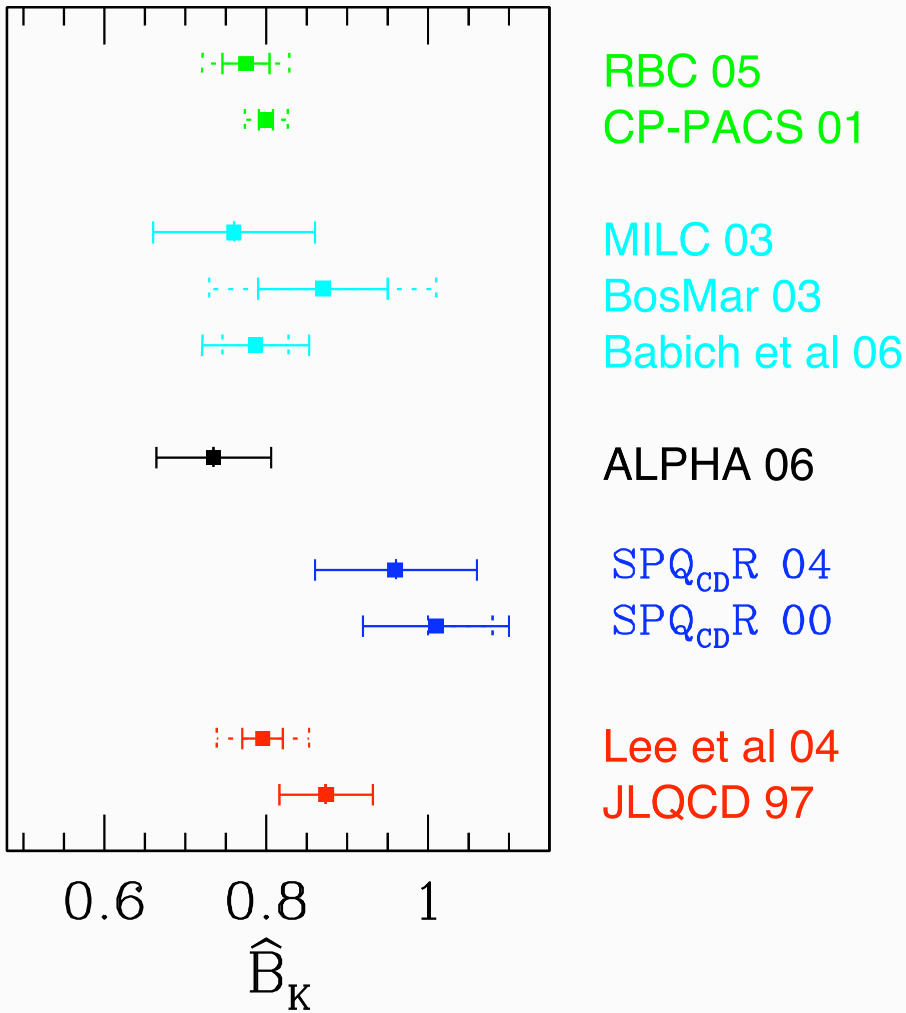


$$\hat{B}_K = 0.735(71)$$

$$\bar{B}_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.534(52)$$

ALPHA P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V. NPB 776 (2007) 258

Comparison with quenched literature



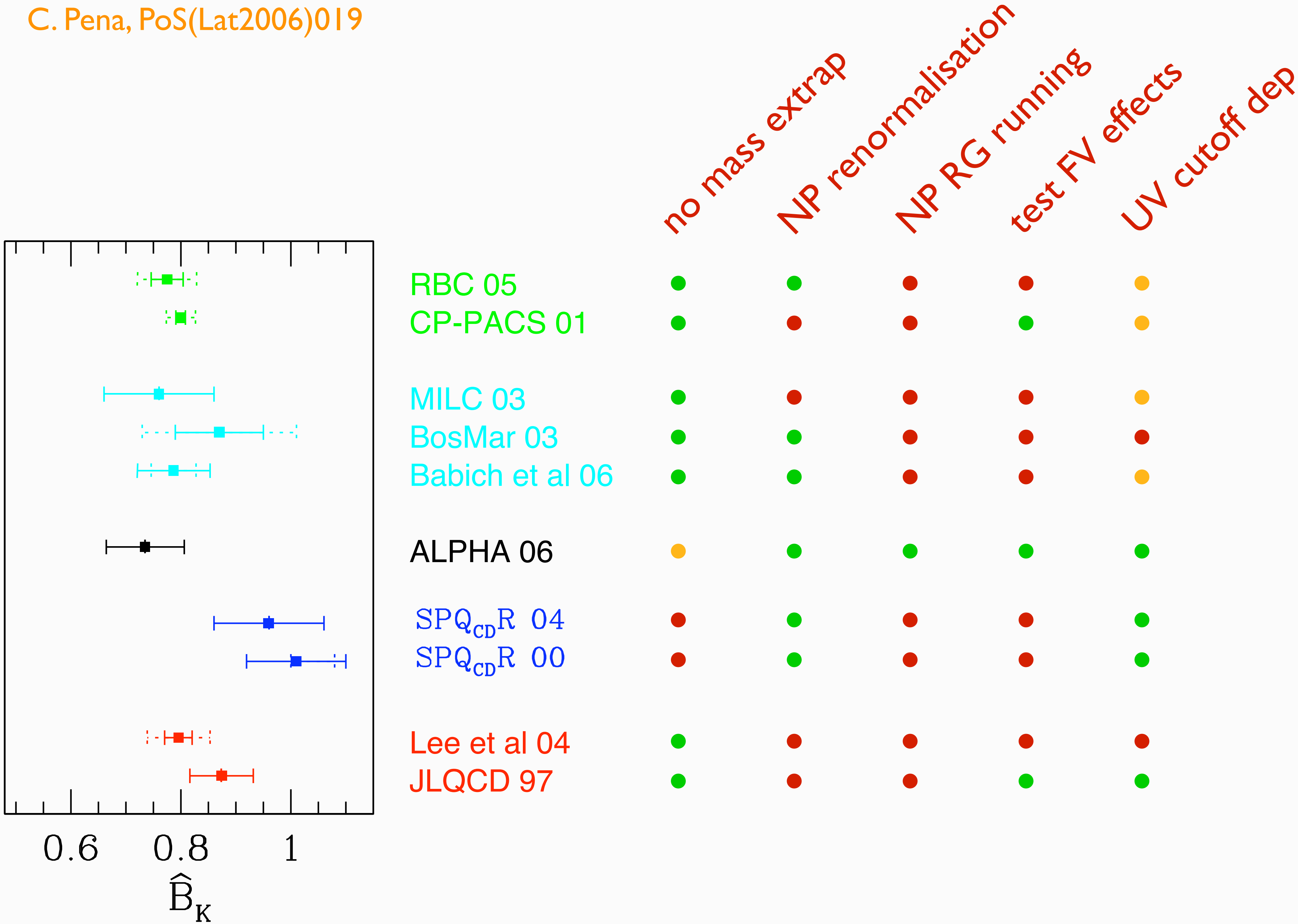
$$\hat{B}_K = 0.735(71)$$

$$\bar{B}_K^{\overline{\mathrm{MS}}}(2\,\mathrm{GeV}) = 0.534(52)$$

Difference with other Wilson fermion computations mainly due to method employed to extract B_K.

Comparison with quenched literature

C. Pena, PoS(Lat2006)019



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$$\hat{B}_K = 0.735(71)$$
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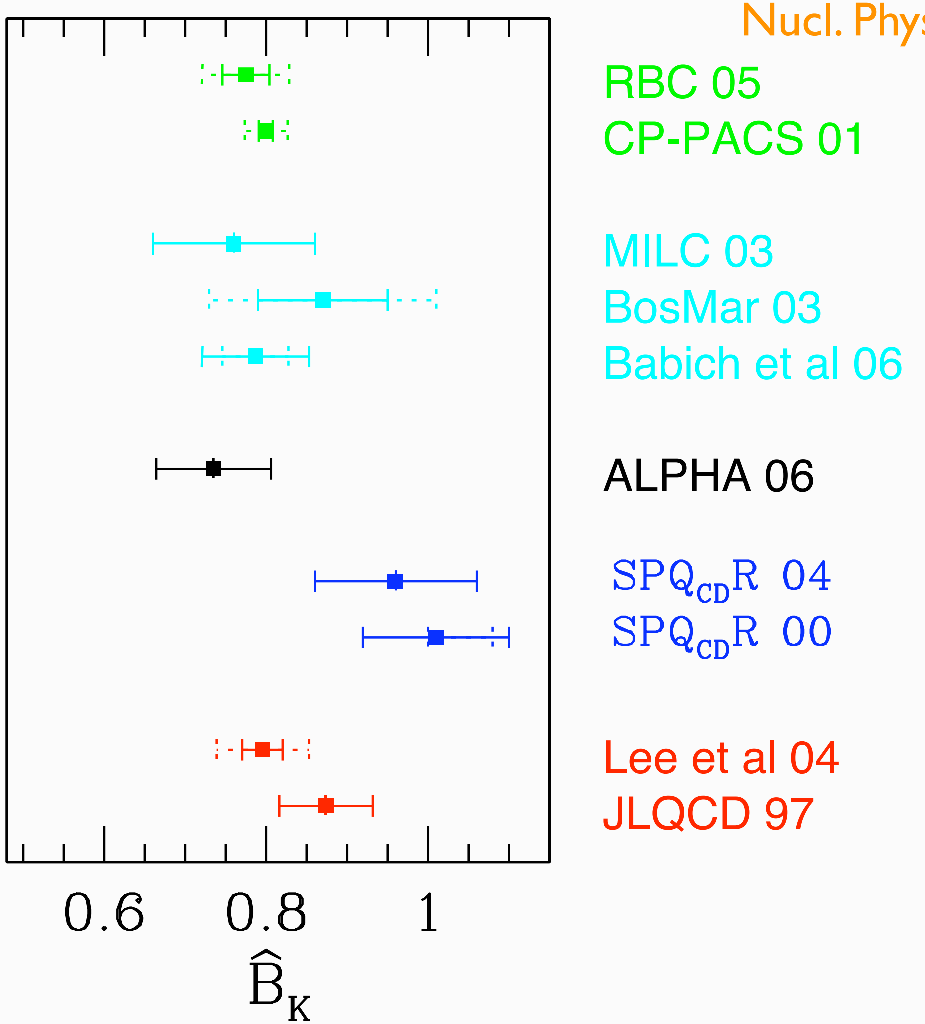
ALPHA P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V.
Nucl. Phys. B776(2007)258



A very precise result, based on DW fermions, has come out recently

CP-PACS Y.Nakamura S.Aoki, Y.Taniguchi, T.Yoshiè
arXiv:0803.2569v2 [hep-lat]

$$\hat{B}_K = 0.782(5)(7)$$
$$\bar{B}_K^{\overline{MS}}(2\mathrm{GeV}) = 0.565(4)(5)$$

tmQCD and renormalization

- Wilson fermions renormalization may be complicated due to loss of chiral symmetry
- this may be simplified in tmQCD through judicious choices of the twist angle
- example IV: $\Delta I = 1/2$ rule (related to $K \rightarrow \pi\,\pi$ transitions)
    - with Wilson fermions, relevant dim=6 operators are subject to power subtractions
    - using tmQCD these "soften" from quadratic to linear (for $K \rightarrow \pi$ transitions)
    - in they quenched approximation they even become finite

C.Pena, S.Sint & A.V. JHEP09(2004)069

- using a variant of tmQCD (Osterwalder/Seiler) they disappear altogether (and improvement is saved) at the cost of having different regularizations in the valence and sea sector

R.Ferezzotti & G.C. Rossi JHEP10(2004)070

3rd advantage:
automatic improvement

R. Frezzotti, G.C. Rossi, JHEP08 (2004) 007

Automatic improvement

- NB: the fully twisted case $\omega = \pi/2$ is of particular interest. Classically we have:

$$\mathcal{L} = \bar{\psi}\left[\not{D} + M\ \exp[i\frac{\pi}{2}\tau^3\gamma_5]\right]\psi = \bar{\psi}\left[\not{D} + i\,\mu\,\tau^3\,\gamma_5\right]\psi$$

- the lattice version requires introduction of the Wilson term but also of the critical standard mass m_{cr} in order to ensure full twist

$$\mathcal{L} = \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + m_{\rm cr} + i\mu_q\tau^3\gamma_5\right]\psi$$

- NB: the concept of twist angle at the quantum level requires renormalized masses:

$$\tan(\omega) = \frac{\mu_{\rm R}}{m_{\rm R}} = \frac{Z_\mu\,\mu_q}{Z_m\,[m_0 - m_{\rm cr}]}$$

$$\omega = \frac{\pi}{2} \leftrightarrow m_0 = m_{\rm cr}$$

Automatic improvement

- recall twisted vector symmetry (isospin) is ***$SU_V(2)_\omega$*** broken to ***$U_V^3(1)$*** by Wilson term

$$\mathcal{L} \quad = \quad \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) \; + \; \frac{1}{2}ar\nabla_\mu^*\nabla_\mu \; + \; m_{\rm cr} \; + \; i\mu_q\tau^3\gamma_5 \;\right]\psi$$

- at $\omega = \pi/2$ different subgroups of ***$SU_V(2)_{\pi/2}$*** remain unbroken by either the (twisted) mass term or the Wilson term
- ***$U_V^1(1)$*** and ***$U_V^2(1)$*:** unbroken by μ-mass term; hard-broken by Wilson and **m_{cr}**-terms

$$\psi \quad \rightarrow \quad \exp\left[i\frac{\alpha^1}{2}\gamma_5\tau^2\right]\,\psi$$

$$\bar{\psi} \quad \rightarrow \quad \bar{\psi}\;\exp\left[i\frac{\alpha^1}{2}\gamma_5\tau^2\right]$$

- this is ***$U_V^1(1)$*** ; similarly for ***$U_V^2(1)$***
- i.e. **vector** symmetry in fully tmQCD has an **axial** form
- it is a **vector** (flavour) symmetry as it is preserved by the mass term

Automatic improvement

- recall twisted vector symmetry (isospin) is ***$SU_V(2)_\omega$*** broken to ***$U_V^3(1)$*** by Wilson term

$$\mathcal{L} \quad = \quad \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) \; + \; \frac{1}{2}ar\nabla^*_\mu\nabla_\mu \; + \; m_{\rm cr} \; + \; i\mu_q\tau^3\gamma_5 \;\right]\psi$$

- at $\omega = \pi/2$ different subgroups of ***$SU_A(2)_{\pi/2}$*** remain unbroken by either the (twisted) mass term or the Wilson term
- ***$U_A^1(1)$*** and ***$U_A^2(1)$*:** softly-broken by μ-mass term; unbroken by Wilson and **m_{cr}**-terms

$$\psi \quad \rightarrow \quad \exp\left[i\frac{\alpha^1}{2}\tau^2\right]\,\psi$$

$$\bar{\psi} \quad \rightarrow \quad \bar{\psi}\;\exp\left[-\,i\frac{\alpha^1}{2}\tau^2\right]$$

- this is ***$U_A^1(1)$*** ; similarly for ***$U_A^2(1)$***
- i.e. **axial** symmetry in fully tmQCD has an **vector** form
- it is a **axial** symmetry as it is restored at vanishing mass (in the continuum)

- NB: in the fully tmQCD the μ-mass term and the Wilson/m_{cr}-terms transform "orthogonally"

Automatic improvement

- the study of discretization effects of lattice observables is based on the Symanzik expansion
- the lattice action close to the continuum is described in terms of an effective theory

$$\mathcal{S}_{\rm Latt} = \mathcal{S}_0 + a\mathcal{S}_1 + \cdots = \int d^4y\, \mathcal{L}_0 + \int d^4y\, \mathcal{L}_1 + \cdots$$

- for the fully twisted lattice action

$$\mathcal{L} = \bar\psi\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + m_{\rm cr} + i\mu_q\tau^3\gamma_5\right]\psi$$

- the Symanzik expansion counter-terms are:

$$\begin{aligned}\mathcal{L}_0 &= \bar\psi\left[\not{D} + i\,\mu_{\rm R}\,\tau^3\,\gamma_5\right]\psi \\ \mathcal{L}_1 &= i\,c_{\rm sw}\,(\bar\psi\,\sigma\cdot F\,\psi) + c_\mu\,\mu_{\rm R}^2\,(\bar\psi\psi)\end{aligned}$$

- the Symanzik expansion for a lattice field operator is:

$$\Phi_{\rm Latt} = \Phi_0 + a\,\Phi_1 + \cdots$$

Automatic improvement

- the study of discretization effects of lattice observables is based on the Symanzik expansion
- the lattice action close to the continuum is described in terms of an effective theory

$$\mathcal{S}_{\rm Latt} = \mathcal{S}_0 + a\mathcal{S}_1 + \cdots = \int d^4y\, \mathcal{L}_0 + \int d^4y\, \mathcal{L}_1 + \cdots$$

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$$\mathcal{L} = \bar\psi \left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + m_{\rm cr} + i\mu_q\tau^3\gamma_5\right]\psi$$

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dimension d

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$$\mathcal{S}_{\rm Latt} = \mathcal{S}_0 + a\mathcal{S}_1 + \cdots = \int d^4y\, \mathcal{L}_0 + \int d^4y\, \mathcal{L}_1 + \cdots$$

- for the fully twisted lattice action

$$\mathcal{L} = \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) + \frac{1}{2}ar\nabla^*_\mu\nabla_\mu + m_{\rm cr} + i\mu_q\tau^3\gamma_5\right]\psi$$

- the Symanzik expansion counter-terms are:

$$\mathcal{L}_0 = \bar{\psi}\left[\not{D} + i\,\mu_{\rm R}\,\tau^3\,\gamma_5\right]\psi$$

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- the Symanzik expansion for a lattice field operator is:

$$\Phi_{\rm Latt} = \Phi_0 + a\,\Phi_1 + \cdots$$

dimension d

dimension d+1

Automatic improvement

- automatic improvement is based on the following field transformations
- discrete "chiral" transformations R_5^1 :
- lattice action terms tranform as R_5^1 eigenstates

$$\psi \quad \rightarrow \quad i\,\gamma_5\,\tau^1\,\psi$$
$$\bar{\psi} \quad \rightarrow \quad \bar{\psi}\,i\,\gamma_5\,\tau^1$$

$$\mathcal{L} \quad = \quad \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*) \; + \; \frac{1}{2}ar\nabla_\mu^*\nabla_\mu \; + \; m_{\mathrm{cr}} \; + \; i\mu_q\tau^3\gamma_5\,\right]\psi$$

$\mathcal{L}-\mathrm{terms}$	R_5^1		
$\bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla_\mu^*)\right]\psi$	$+$		
$\bar{\psi}\left[\frac{1}{2}ar\nabla_\mu^*\nabla_\mu\,\right]\psi$	$-$		
$\bar{\psi}\,m_{\mathrm{cr}}\psi$	$-$		
$\bar{\psi}\left[\,i\mu_q\tau^3\gamma_5\,\right]\psi$	$+$		

Automatic improvement

- automatic improvement is based on the following field transformations
- operator dimensionality transformations *D* :
- lattice action terms tranform as *D* eigenstates

$$
\begin{array}{rcl}
U_\mu(x) & \rightarrow & U_\mu^\dagger(-x-a\hat{\mu}) \\
\psi(x) & \rightarrow & \exp[3i\pi/2]\,\psi(-x) \\
\bar{\psi}(x) & \rightarrow & \bar{\psi}(-x)\exp[3i\pi/2]
\end{array}
$$

$$
\mathcal{L} \quad = \quad \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla_\mu^*) \;+\; \frac{1}{2}ar\nabla_\mu^*\nabla_\mu \;+\; m_{\mathrm{cr}} \;+\; i\mu_q\tau^3\gamma_5\right]\psi
$$

$\mathcal{L}-\mathrm{terms}$	R_5^1	$\mathcal{D}$	
$\bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla_\mu^*)\right]\psi$	$+$	$+$	
$\bar{\psi}\left[\frac{1}{2}ar\nabla_\mu^*\nabla_\mu\right]\psi$	$-$	$-$	
$\bar{\psi}\,m_{\mathrm{cr}}\psi$	$-$	$-$	
$\bar{\psi}\left[i\mu_q\tau^3\gamma_5\right]\psi$	$+$	$-$	

Automatic improvement

- automatic improvement is based on the following field transformations
- operator dimensionality transformations *D* :
- lattice action terms tranform as *D* eigenstates

$$\begin{array}{rcl} U_\mu(x) & \rightarrow & U_\mu^\dagger(-x-a\hat{\mu}) \\ \psi(x) & \rightarrow & \exp[3i\pi/2]\,\psi(-x) \\ \bar{\psi}(x) & \rightarrow & \bar{\psi}(-x)\exp[3i\pi/2] \end{array}$$

$$\mathcal{L} = \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla_\mu^*) + \frac{1}{2}ar\nabla_\mu^*\nabla_\mu + m_{\rm cr} + i\mu_q\tau^3\gamma_5\right]\psi$$

$\mathcal{L}-$ terms	R_5^1	$\mathcal{D}$	
$\bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu+\nabla_\mu^*)\right]\psi$	+	+	
$\bar{\psi}\left[\frac{1}{2}ar\nabla_\mu^*\nabla_\mu\right]\psi$	−	−	
$\bar{\psi}\, m_{\rm cr}\psi$	−	−	
$\bar{\psi}\left[i\mu_q\tau^3\gamma_5\right]\psi$	+	−	

- NB: massless theory invariant under $R_5^1 \otimes D$

Automatic improvement

- automatic improvement is based on the following field transformations
- twisted mass sign flip $\mu \to -\mu$
- lattice action terms tranform as *sign flip* eigenstates

$$\mathcal{L} \quad = \quad \bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu) \;+\; \frac{1}{2}ar\nabla^*_\mu\nabla_\mu \;+\; m_{\mathrm{cr}} \;+\; i\mu_q\tau^3\gamma_5\right]\psi$$

$\mathcal{L}-\mathrm{terms}$	R_5^1	$\mathcal{D}$	$\mu \to -\mu$
$\bar{\psi}\left[\frac{1}{2}\gamma_\mu(\nabla_\mu + \nabla^*_\mu)\right]\psi$	+	+	+
$\bar{\psi}\left[\frac{1}{2}ar\nabla^*_\mu\nabla_\mu\right]\psi$	−	−	+
$\bar{\psi}\, m_{\mathrm{cr}}\psi$	−	−	+
$\bar{\psi}\left[i\mu_q\tau^3\gamma_5\right]\psi$	+	−	−

- NB: massive theory invariant under $R_5^1 \otimes D \otimes [\mu \to -\mu]$

Automatic improvement

- the lowst-order Symanzik expansion for the vev of an operator Φ is:

$$< \Phi > = < \Phi_0 >_0 \; + a \; < \Phi_1 >_0 \; - a \int d^4 y < \Phi_0 \; \mathcal{L}_1 >_0$$

- the LHS is a lattice vev, determined by the lattice fully twisted action
- for an operator Φ with positive $R_5{}^1$ parity and even dimension d, the LHS is invariant under $R_5{}^1 \otimes D \otimes [\mu \to -\mu]$
- also the RHS must be invariant under $R_5{}^1 \otimes D \otimes [\mu \to -\mu]$
- the RHS operators and vev are continuum quantities determined by the continuum tmQCD action with positive $R_5{}^1$ parity, $\mathcal{L}_0$

$$\begin{aligned} \mathcal{L}_0 &= \bar{\psi} \left[\not{D} \; + \; i \, \mu_{\mathrm{R}} \, \tau^3 \, \gamma_5 \right] \psi \\ \mathcal{L}_1 &= i \, c_{\mathrm{sw}} \; (\bar{\psi} \; \sigma \cdot F \; \psi) \; + \; c_\mu \; \mu_{\mathrm{R}}^2 \; (\bar{\psi} \psi) \end{aligned}$$

- the lattice operator Φ_0 has positive $R_5{}^1$ parity and even dimension d
- the lattice operator Φ_1 has odd dimension $d+1$ and therefore negative $R_5{}^1$ parity
- thus $< \Phi_1 >_0$ vanishes as it is $R_5{}^1$ odd, weighted by an $R_5{}^1$ even action $\mathcal{L}_0$

Automatic improvement

- the lowst-order Symanzik expansion for the vev of an operator Φ is:

$$< \Phi > = < \Phi_0 >_0 \; + a \; < \Phi_1 >_0 \; -a \int d^4y < \Phi_0 \; \mathcal{L}_1 >_0$$

- the LHS is a lattice vev, determined by the lattice fully twisted action
- for an operator Φ with positive $R_5{}^{\mathrm{I}}$ parity and even dimension *d*, the LHS is invariant under $R_5{}^{\mathrm{I}} \otimes D \otimes [\mu \rightarrow -\mu]$
- also the RHS must be invariant under $R_5{}^{\mathrm{I}} \otimes D \otimes [\mu \rightarrow -\mu]$
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$$\begin{aligned} \mathcal{L}_0 \quad &= \quad \bar{\psi} \left[\not{D} \; + \; i \, \mu_{\mathrm{R}} \, \tau^3 \, \gamma_5 \; \right] \psi \\ \mathcal{L}_1 \quad &= \quad i \, c_{\mathrm{sw}} \; (\bar{\psi} \; \sigma \cdot F \; \psi) \; + \; c_\mu \; \mu_{\mathrm{R}}^2 \; (\bar{\psi} \psi) \end{aligned}$$

- the continuum *O*(a) counterterm of the action is $\mathcal{L}_1$, with negative $R_5{}^{\mathrm{I}}$ parity
- thus $< \Phi_0 \mathcal{L}_1 >_0$ vanishes as it is $R_5{}^{\mathrm{I}}$ odd, weighted by an $R_5{}^{\mathrm{I}}$ even action $\mathcal{L}_0$
- fully twisted QCD is not improved, but has automatically improved vev !!!!!

Automatic improvement

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- NB (subtlety): the proof rests on the vanishing of the continuum vev

$$< \Phi_0 \mathcal{L}_1 >_0 \qquad\qquad < \Phi_1 >_0$$

- they vanish due to their breaking of the discrete "chiral" symmetry R_5^I, which is a symmetry of the continuum tmQCD $\mathcal{L}_0$
- BUT: is this true, i.e. do these "chiral condensates" vanish in a theory with SSB?
- YES! because the term generating SSB is the twisted mass term, while O(a) counter-tems are generated by the "chirally orthogonal" Wilson term
- stated differently, the discrete "chiral symmetry" R_5^I is a specific **vector** rotation ***$U_V^2(I)$***, which does not generate SSB

$$\begin{aligned} \mathcal{L}_0 &= \bar\psi \left[\not{D} \; + \; i\,\mu_{\mathrm{R}}\,\tau^3\,\gamma_5 \right] \psi \\ \mathcal{L}_1 &= i\,c_{\mathrm{sw}}\,(\bar\psi\;\sigma\cdot F\;\psi) \; + \; c_\mu\;\mu_{\mathrm{R}}^2\;(\bar\psi\psi) \end{aligned}$$

Automatic improvement

- the lowst-order Symanzik expansion for the vev of an operator Φ is:

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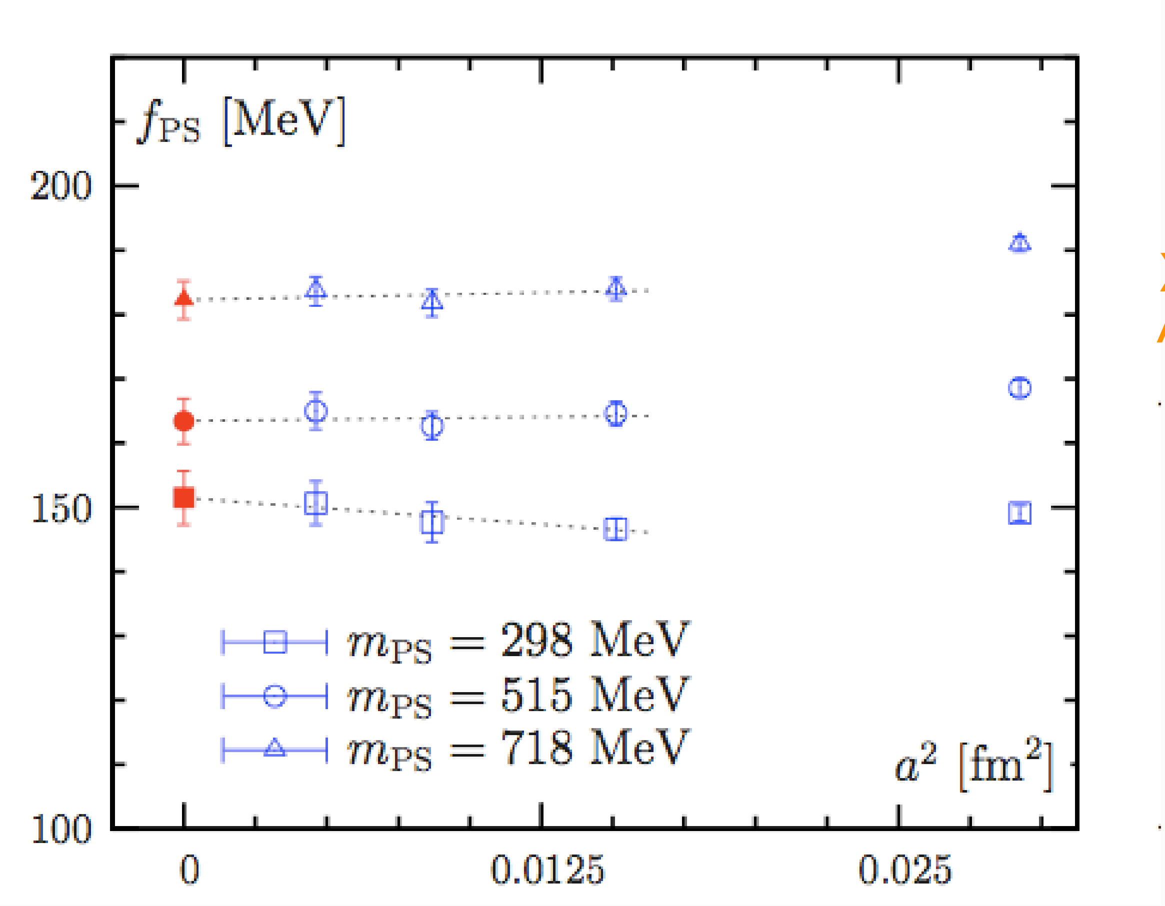
- stated differently, the discrete “chiral symmetry” R_5^I is a specific **vector** rotation $U_V^2(I)$, which does not generate SSB
- NB: in simulations we must take extra care that the continuum limit is approached before the chiral limit; the chiral phase of the vacuum must be driven by the mass term and not by the Wilson term

$$\mu \ > \ a\Lambda^2_{\rm QCD}$$

Automatic improvement

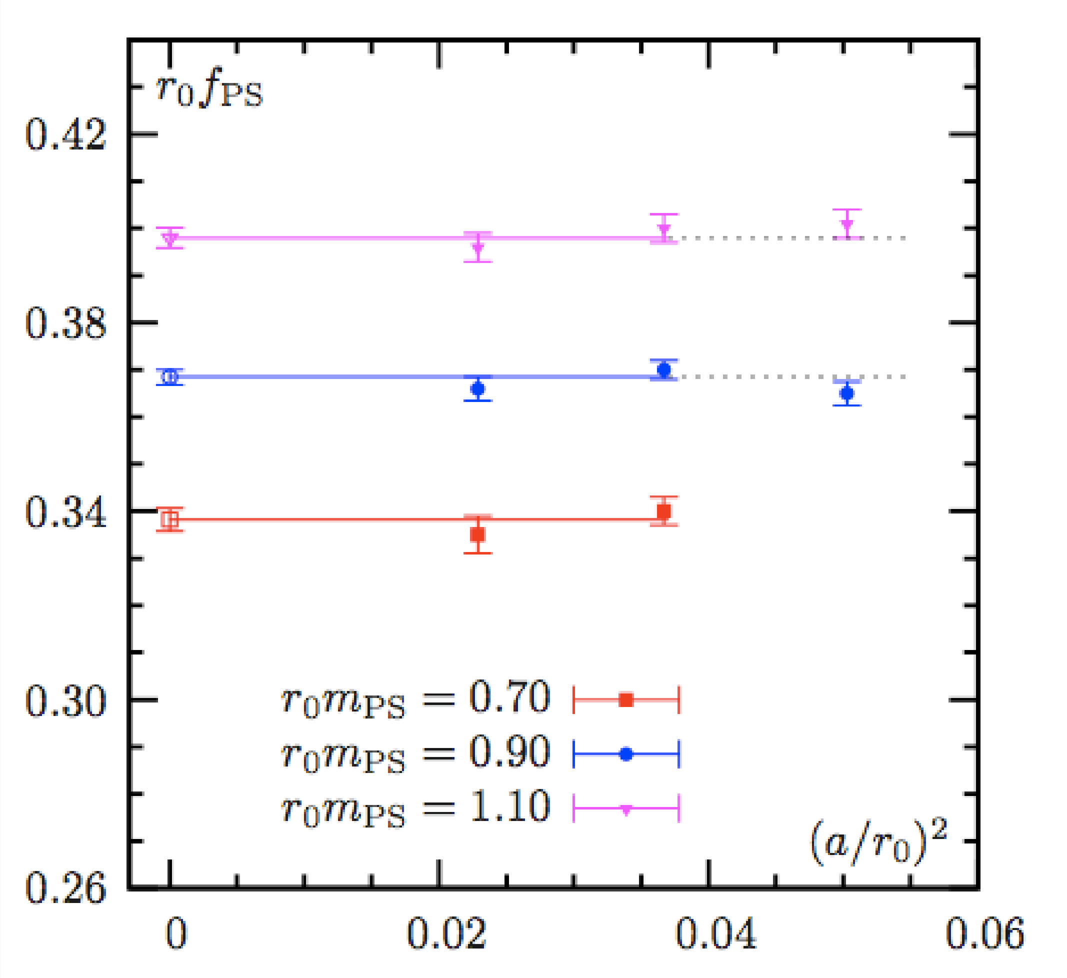
- numerical evidence supports automatic improvement
- result shown is quenched



χLF Collab. K. Jansen, M. Papiutto, A. Shindler, C. Urbach, I. Wetzorke JHEP09 (2005) 071

Automatic improvement

- numerical evidence supports automatic improvement
- result shown is unquenched $N_f = 2$



ETMC Collab., C. Urbach, PoS(LAT2007) 022

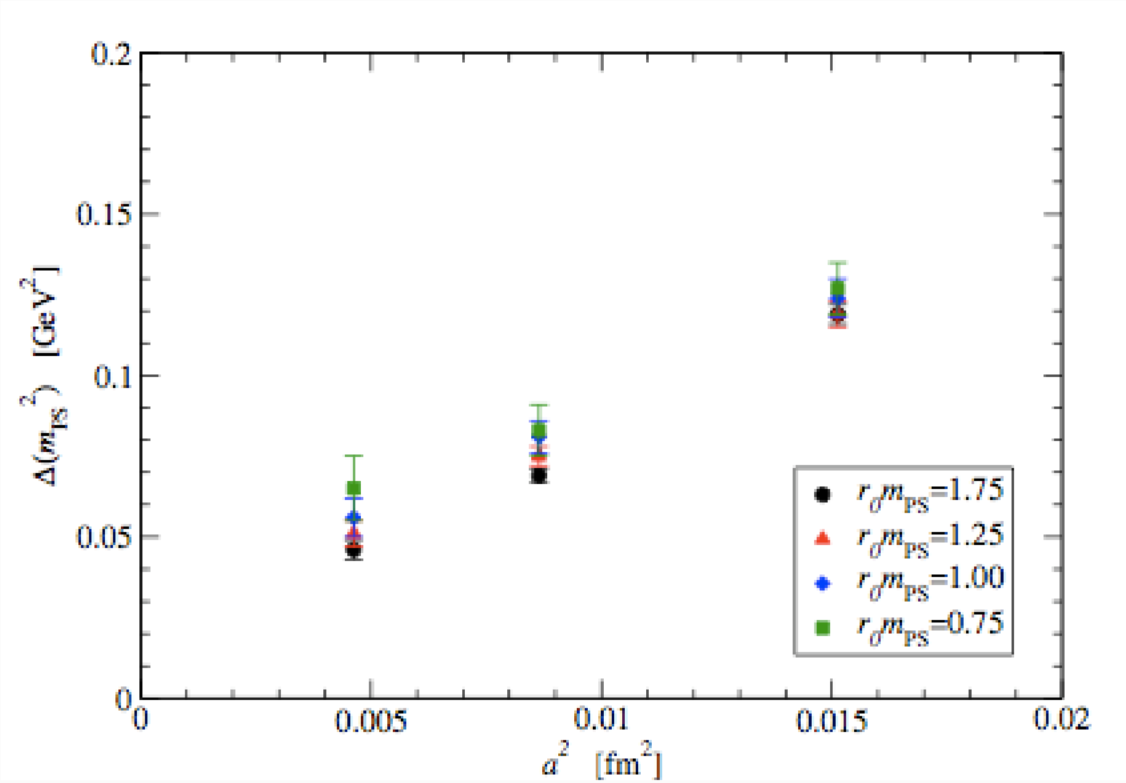
Shortcoming: loss of flavour symmetry

Flavour symmetry violation

- it generates mass splittings between, say, neutral and charged pions
- it is a discretization effect, which should vanish in the continuum
- very preliminary studies suggest that this is a big effect in the quenched approximation, but diminishes significantly in the $N_f = 2$ case
- the quenched case offers an interesting play-field for explorative studies of flavour breaking, as it is possible (and cheap) to compare, on the same ensemble, mesons with the following valence quark content:
    - 2 twisted flavours
    - 2 untwisted flavours (the standard Wilson case, without flavour breaking)
    - 1 twisted and 1 untwisted flavour
- further detailed unquenched studies are required
- the main difficulty is an accurate and efficient computation of disconnected diagrams for neutral pions; cf. C. Michael et al in various recent conference proceedings

Flavour symmetry violation

- mass splittings between, say, neutral and charged Kaons
- 4 quenched flavours organized in two maximally twisted doublets
- K^0 made of an “up” and a “down” twisted flavour
- K^+ made of two “up” twisted flavours
- at the smallest lattice spacing, neutral-charged Kaon splitting is $m_{K0} - m_{K+} \sim$ 50 MeV



A.M. Abdel-Rehim, R. Lewis, R.M. Woloshyn, J.M.S. Wu, Phys.Rev.D74(2006)014507

Flavour symmetry violation

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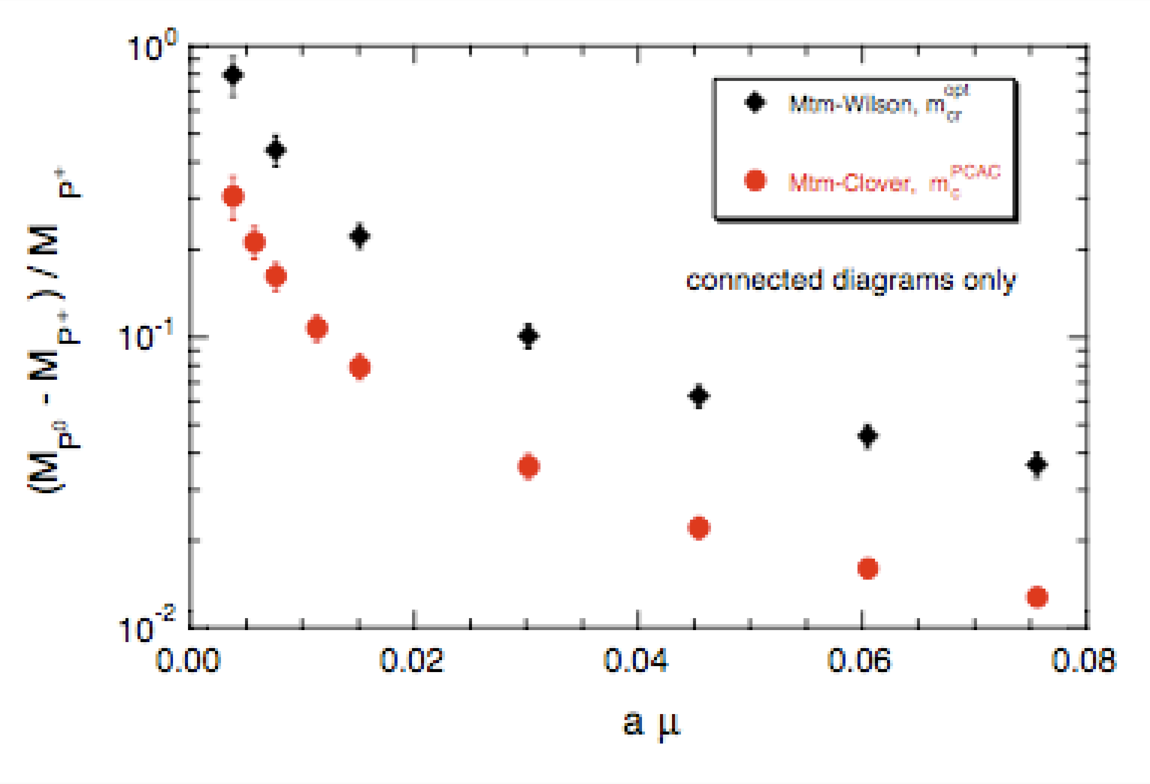
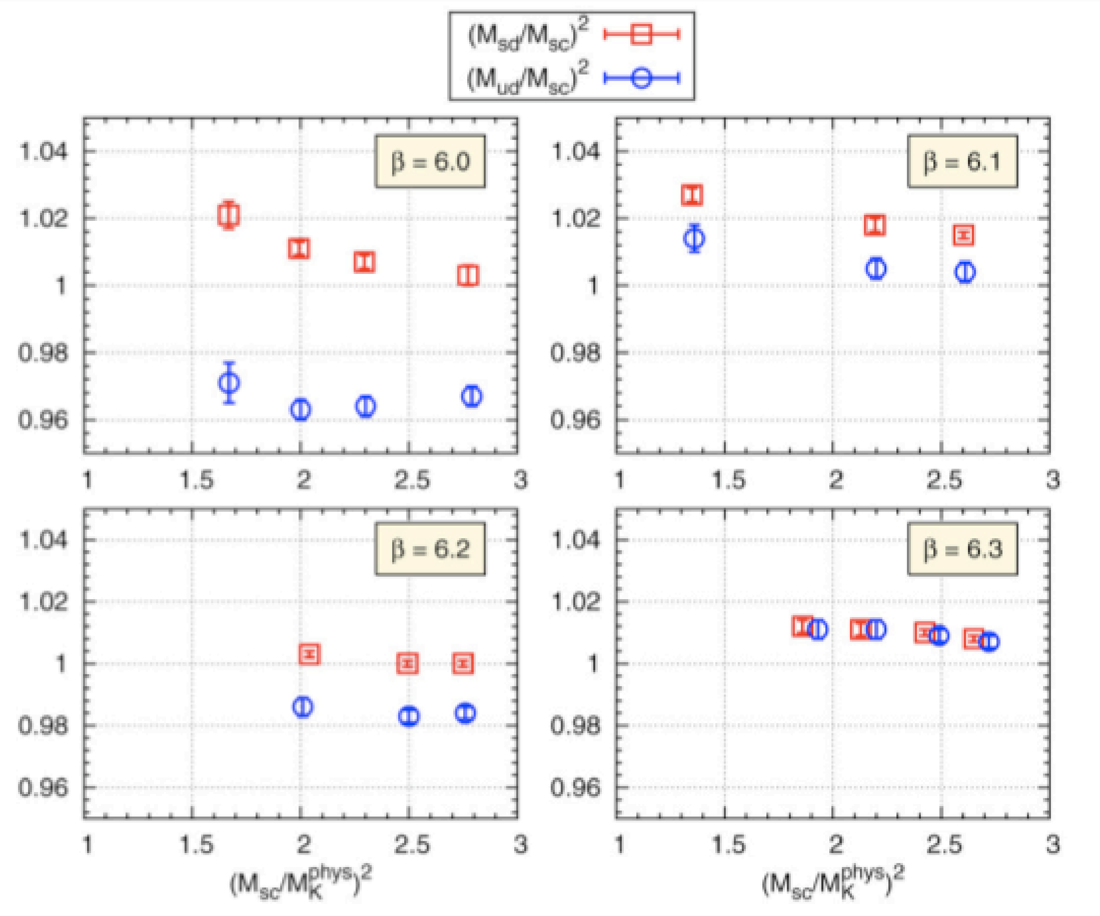
- 4 quenched flavours organized in two maximally twisted doublets
- π^0 made of an "up" and a "down" twisted flavour
- π^+ made of two "up" twisted flavours
- comparison between Wilson/Clover discretizations, at a singe lattice spacing, for very small quark masses



D. Bećirević Ph. Boucaud, V.Lubicz, G.Martinelli, F.Mescia, S.Simula, C.Tarantino
Phys.Rev.D74(2006)034501

Flavour symmetry violation

- mass splittings between, say, neutral and charged Kaons
- 4 quenched flavours organized in a maximally twisted and an untwisted doublet
- pseudoscalars made of twisted (u-d), untwisted (s-c) and mixed (s-d) valence quarks
- NB: the untwisted (s-c) case is the standard Wilson one, without flavour breaking
- comparison at a several lattice spacings, for very quark masses "above" strangeness



ALPHA P.Dimopoulos, J.Heitger, F.Palombi, C.Pena, S.Sint, A.V.
NPB 776 (2007) 258

Flavour symmetry violation

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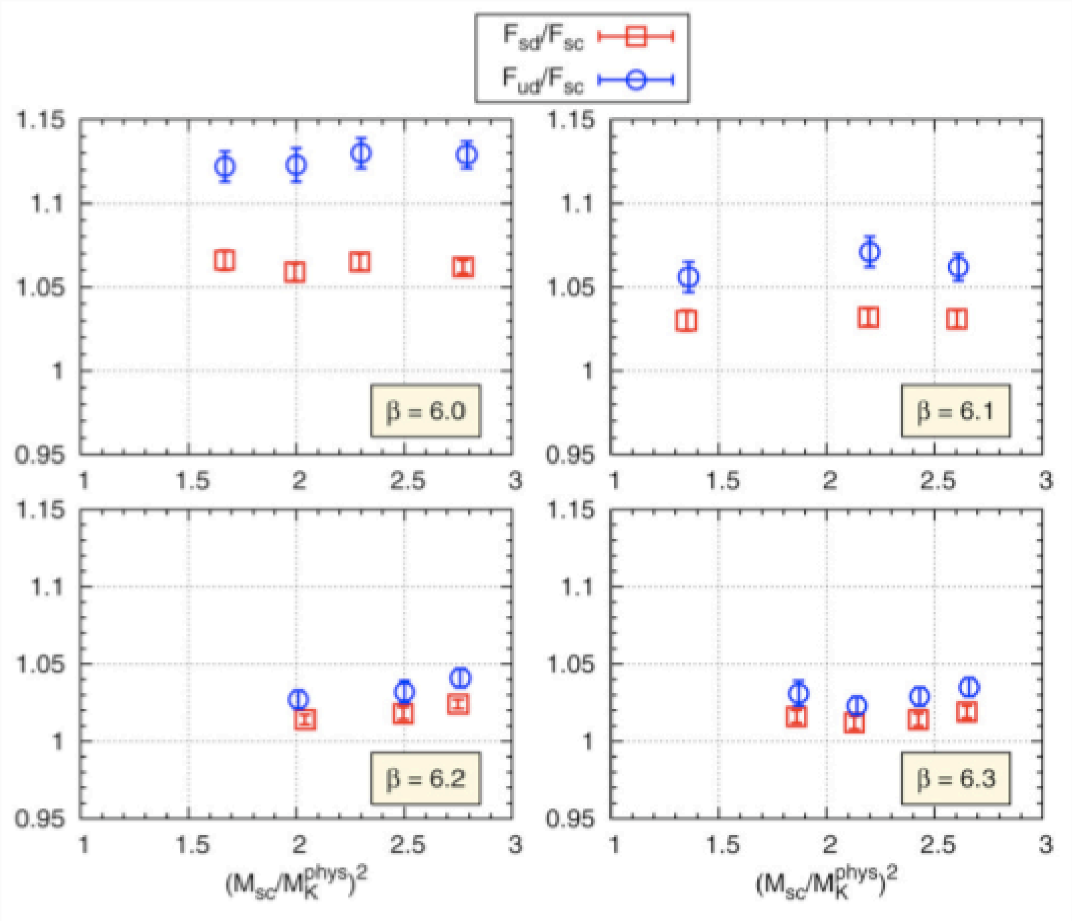

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NPB 776 (2007) 258

Selected ETMC results

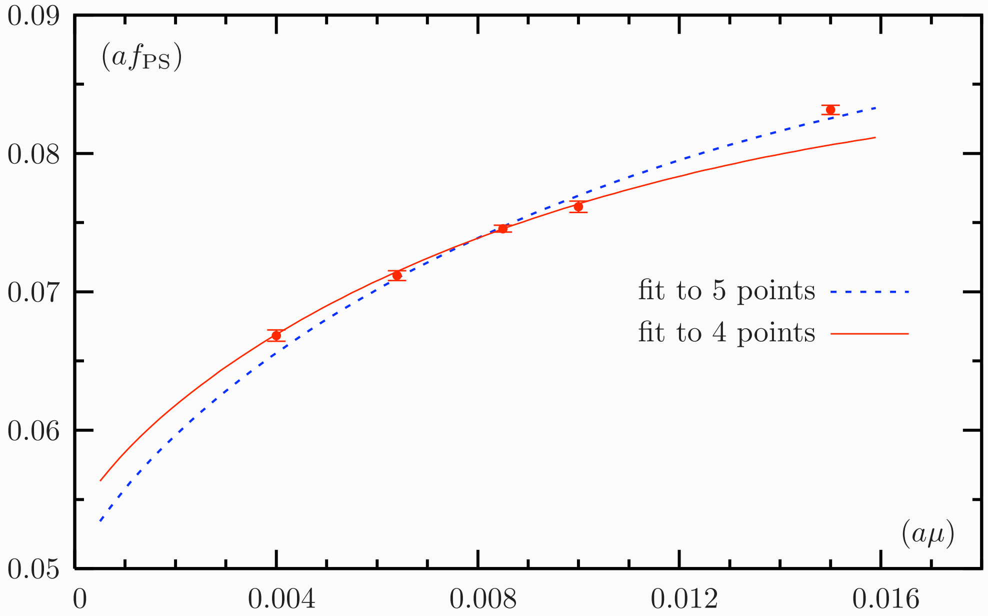
Selected ETMC results

Using tmQCD fermions with $N_f = 2$ the ETM-Collaboration reports:

ETMC Ph. Boucaud et al. Phys.Lett. B650 (2007) 304

- one lattice spacing $a \sim 0.1$ fm; one lattice volume $L \sim 2.4$ fm
- light quark masses (sea) 300 MeV < m_π < *550 MeV*
- no axial current (and no Z_A) is needed, due to tmQCD Ward identity
- due to tmQCD @ twist angle $\alpha = \pi / 2$, we have automatic *O(a)* improvement

$$f_\chi = 121.3 \pm 0.7$$



Selected ETMC results

Using tmQCD fermions with $N_f = 2$ the ETM-Collaboration reports:

ETMC B. Blossier et al., arXiv:0709.4574v1 [hep-lat]

- one lattice spacing $a \sim 0.1$ fm; two lattice volumes $L \sim 2.4$ fm

$$
\begin{aligned}
m_{ud}^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) &= 3.85 \pm 0.12 \pm 0.40 \ \ \mathrm{MeV} \\
m_{s}^{\overline{\mathrm{MS}}}(2\mathrm{GeV}) &= 105 \pm 3 \pm 9 \ \ \mathrm{MeV} \\
\frac{m_s}{m_{ud}} &= 27.3 \pm 0.3 \pm 1.2 \\
f_K &= 161.7 \pm 1.2 \pm 3.1 \ \ \mathrm{MeV} \\
\frac{F_K}{f_\pi} &= 1.227 \pm 0.009 \pm 0.024
\end{aligned}
$$

Selected ETMC results

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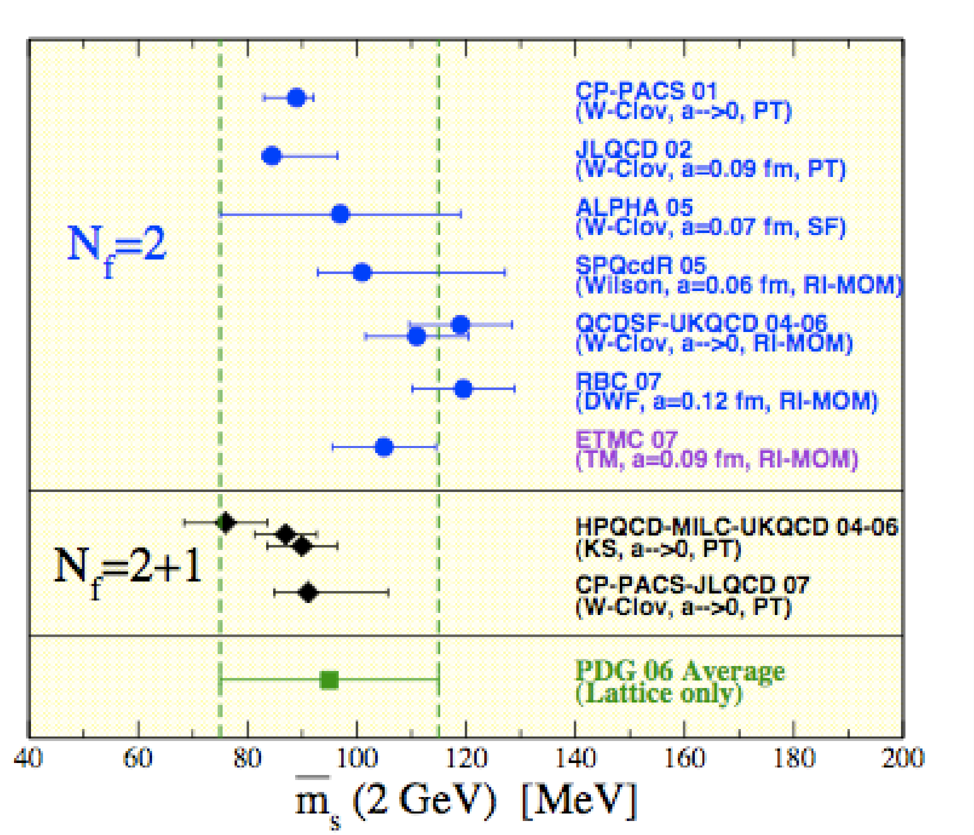
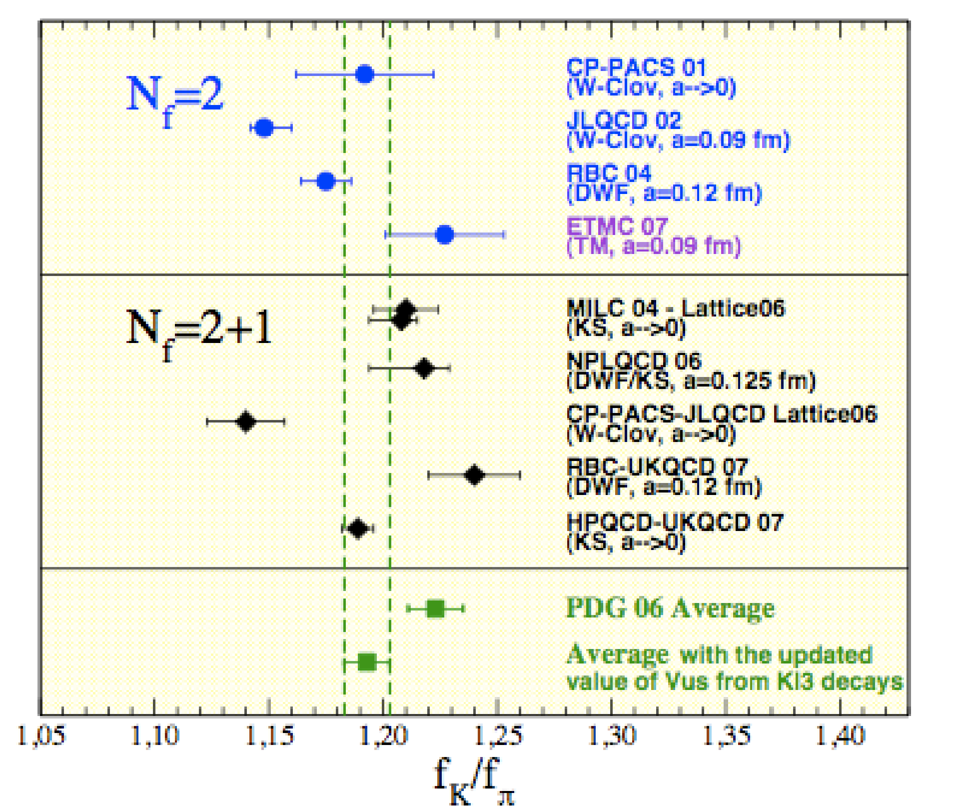
- one lattice spacing $a \sim 0.1$ fm; two lattice volumes $L \sim 2.4 - 3.0$ fm





Concluding remarks

Concuding remarks

- tmQCD has several advantages, making it an efficient alternative to GW-type computations with dynamical fermions; novel results have been obtained at $N_f = 2$
- the simplest quantities (pseudoscalar masses, decay constants, quark masses etc.) are currently being measured, close to the "physical regime" (lightish pions ~ 300MeV)
- it is possible to overcome the limitation of two degenerate flavours in the formalism without losing tmQCD advantages; $N_f = 2+1+1$ simulations are under way
- the main drawback is the lack of flavour symmetry at finite UV cutoff
  - early studies have given a quenched, partilly satisfactory answer
  - detailed unquenched studies are being carried out; the overall qualitative behaviour points out to the vanishing of such effects in the continuum
- one must also be aware that tuning of the theory bare parameters to maximal twist is an issue requiring extra care (not covered here)
- the phase diagramme of tmQCD has also been under study in order to gain further insight to the issues related to symmetry breaking and their restoration in the continuum (not covered here)
- an important theoretical issue is the combination of tmQCD with Schrödinger Functional: S. Sint PoS (LAT2005) 235 (not covered here)