

# PHYSICS OF FLAVOURS AND LATTICE QCD

Spring Course of the International Graduate School, Paris-Bielefeld, GRK 881  
"Quantum Fields and Strongly Interacting Matter"

## Lattice Simulations of Quantum Fields

Laboratoire de Physique Théorique, Université Paris-Sud XI, Orsay, France

March 26 – April 1, 2008

*Block Course for PhD students and postdocs*

### Topics include:

Algorithms and Numerical Methods; QCD with Chirally Invariant Quarks;  
Staggered Quarks and "Fourth Root" Issue; Twisted Mass QCD; Finite Temperature QCD  
Supersymmetry on the Lattice; N-body Simulations in Cosmology

### Lecturers:

Luigi Del Debbio, SUPA Edinburgh & INI Cambridge  
Maarten Golterman, San Francisco State Univ.  
Shoji Hashimoto, KEK Tsukuba  
Michael Joyce, Univ. Paris 6  
David Kaplan, Washington Univ. Seattle & UA Madrid  
Werner Krauth, ENS Paris  
Vittorio Lubicz, Univ. Roma Tre  
Tassos Vladikas, Rome Univ. di Tor Vergata  
Tilo Wettig, Regensburg Univ.  
Edwin Laermann, Bielefeld Univ.  
Philippe Boucaud, Univ. Paris Sud



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For further information: <http://www.th.u-psud.fr/block-course/>

## Vittorio Lubicz



## Laboratoire de Physique Théorique



# 1. FLAVOUR PHYSICS AND ITS MOTIVATIONS

- Introduction to Flavour Physics
- Evidence of physics beyond the Standard Model
- Flavour physics as a probe of New Physics

# 2. FLAVOUR PHYSICS AND LATTICE QCD

- The “precision era” of Lattice QCD (why now)
- $V_{us}$  and the first row unitarity test
- Lattice QCD and the Unitarity Triangle Analysis
- The UTA beyond the Standard Model:  
evidence of New Physics in  $B_s$  mixing
- Lattice QCD and quark masses



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**Appendix:** a simple model of Flavour Physics

# FLAVOUR PHYSICS

## Elementary Particles

Quarks	$u$	$c$	$t$	$\gamma$
	$d$	$s$	$b$	
Leptons	$\nu_e$	$\nu_\mu$	$\nu_\tau$	$Z$
	$e$	$\mu$	$\tau$	

Three Generations of Matter

6 Flavours  
3 Families

- **FLAVOUR**: elementary fermions (matter particles) are 6 flavours of quarks and 6 of leptons

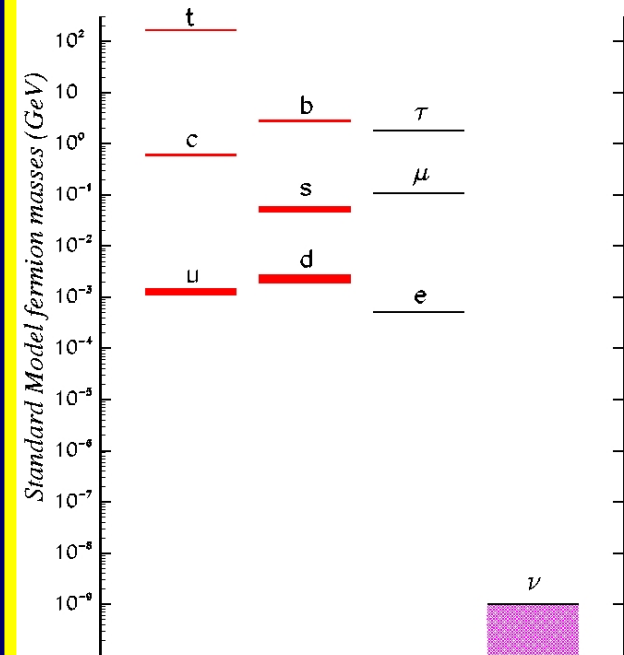
- **MASSES**: Quarks and leptons come in 3 families which only differ for particle masses

- **MIXING**: In the Standard Model flavour is conserved by electromagnetic and strong interactions. Only weak interactions (charged currents) change flavour  $\rightarrow$  CKM matrix and CP violation

# Flavor physics is (well) described but not explained in the Standard Model

A large number of **free parameters** in the flavor sector: **10** parameters in the quark sector ( $6 m_q + 4 \text{ CKM}$ ), **12** in the lepton sector (with massive neutrinos)

- Why **3 families** with their observed particle content?
- Why the **spectrum** of quarks and charged leptons covers 5 orders of magnitude? ( $m_q \sim Yv \sim G_F^{-1/2} \dots$ )
- What give rise to the pattern of **quark mixing** and the magnitude of **CP violation**?



# THE FLAVOUR SYMMETRY:

The Standard Model fermions consist of 3 families with  $(Q_L, L_L, U_R, D_R, E_R)$ .

The largest unitary group that commutes with the gauge group is:

$$G_F = U(3)^5 = [SU(3) \otimes U(1)]^5$$

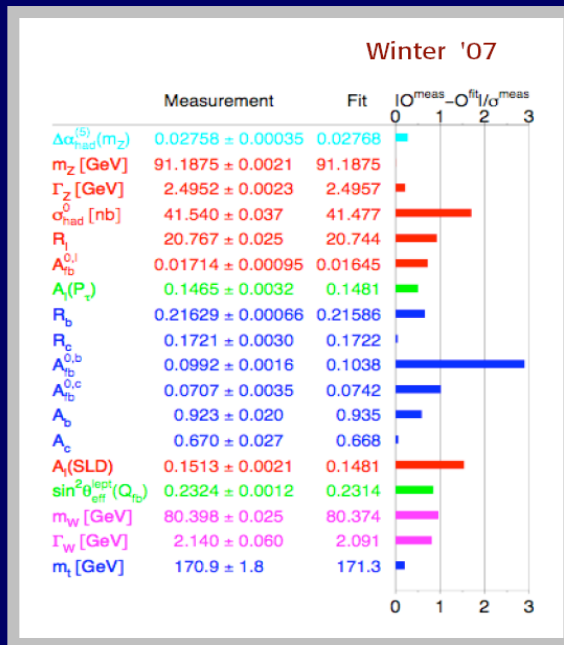
In the SM,  $G_F$  is broken by the Yukawa interactions:

$$G_F \rightarrow U(1)_B \otimes U(1)_L \otimes U(1)_Y$$

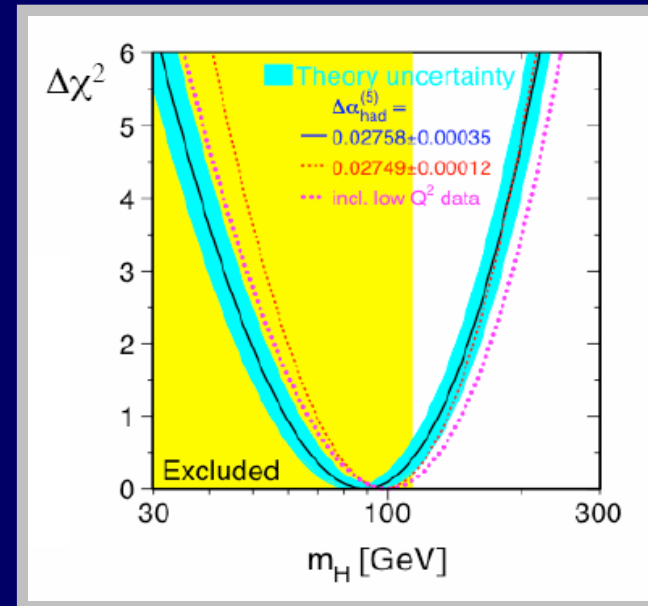
WHICH IS THE ORIGIN OF FLAVOUR SYMMETRY BREAKING?

[Note: also the mechanism of **gauge symmetry breaking** is unknown. A single elementary Higgs doublet is only the simplest solution.]

Experiments show that the **Standard Model with a light Higgs** provides a quite successful description of electro-magnetic, weak and strong interactions (the gauge sector), up to the Fermi scale



$\chi^2/\text{d.o.f.} = 18/13$



$114 \text{ GeV} < m_H < 182 \text{ GeV}$   
@ 95% CL

Despite this success, we have both theoretical and observational evidence of

**PHYSICS BEYOND THE STANDARD MODEL**

**EVIDENCE OF PHYSICS  
BEYOND THE  
STANDARD MODEL**

# PROBLEMS OF THE STANDARD MODEL AND EVIDENCE OF NEW PHYSICS

- **Gravity** ( $M_{\text{Planck}} = (\hbar c/G_N)^{1/2} \approx 10^{19} \text{ GeV}$ )
- **Theory of flavour**
- **Hierarchy** ( $M_{\text{EW}} \ll M_{\text{Planck}}$ )
- **Unification of couplings** ( $M_{\text{GUT}} \approx 10^{15}\text{-}10^{16} \text{ GeV}$ )
- **Neutrino masses** ( $M \approx M_{\text{GUT}}$ )
- **Dark matter** ( $\Omega_M \approx 0.3$ )
- **Vacuum energy** ( $\Omega_\Lambda \approx 0.7$ )
- **Baryogenesis**
- **Inflation** ( $\Omega_{\text{tot}} = 1, \dots$ )

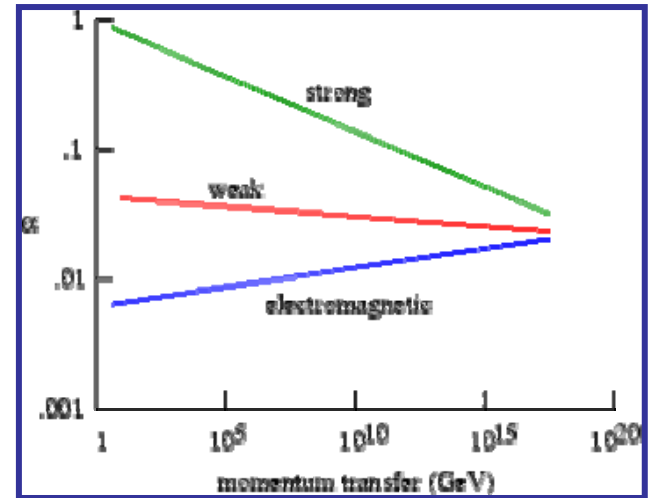
**Clash with the  
Standard Model  
of cosmology**



# UNIFICATION OF COUPLINGS

The running of gauge couplings provides **strong indication of unification**.

Precise unification, however, **fails in the SM** [  $\alpha_s(M_Z) \approx 0.073$  ]  
(compatible with low energy SUSY)



**GUT are very appealing for several reasons**

- Unity of forces
- Unity of quark and leptons (different directions in  $G$ )
- Family Q-numbers (in  $SO(10)$  a whole family in 16)
- Charge quantization ( $Q_d = -1/N_c = -1/3$ )
- B and L non conservation

# NEUTRINO MASSES

The existence of neutrino masses and mixings is well established. But neutrinos are massless in the SM and the simple extension of the SM with the inclusion of  $\nu_R$  looks very unnatural.

**Neutrino masses are really special:  $m_t / (\Delta m_{\text{atm}}) \sim 10^{12}$**

A natural solution:  $\nu$ 's are Majorana particles and get masses through L violating interactions suppressed by a large scale  $M$

$$O_5 = \frac{(HL)^T \lambda (HL)}{M} + h.c.$$

The seesaw mechanism is a specific realization

$$m_\nu \sim \frac{m^2}{M}$$

For  $m_\nu \sim 0.05 \text{ eV}$  and  $m \sim \nu \sim 200 \text{ GeV}$   $\rightarrow$

$$M \sim 10^{15} \text{ GeV} \sim M_{\text{GUT}}$$

# DARK MATTER

$$\Omega_{\text{tot}} = \Omega_{\text{vac}} + \Omega_{\text{mat}} + \Omega_{\text{rad}} + \dots = 1.005(6)$$

Inflation:  $\Omega_{\text{tot}} = 1$   
Flat Universe

$$\Omega_{\text{vac}} \approx 0.7, \quad \Omega_{\text{mat}} \approx 0.3 \quad \text{Both problematic!}$$

$$(\Omega_{\text{rad}} \approx 10^{-5})$$

$$\Omega_{\text{mat}} = \Omega_{\text{b}} + \Omega_{\text{dm}}$$

Baryonic matter

Dark matter (i.e. non-luminous and non-absorbing)

$$\Omega_{\text{b}} \approx 0.05, \quad \Omega_{\text{dm}} \approx 0.23$$

More than 80% of matter is **non baryonic** dark matter !!

**NO CANDIDATES IN THE SM.  $\nu$ 'S ARE NOT RELEVANT FOR DM**

Most of DM should be cold



All hot DM would have not permitted galaxies to form

**(Cold) Hot DM  $\equiv$  (Non) relativistic** at the onset of galaxy formation

WIMP (LSP:neutralino, Extra-d.:LKP,...),  
SWIMP (gravitino), axions, axinos,...

**Many NP candidates**  
Remarkably:  $M_{\text{WIMP}} \sim M_{\text{EW}}$

# VACUUM ENERGY

$$\Omega_{\text{vac}} \approx 0.7$$

The scale of the cosmological constant is a big mystery

- In **QFT** the **energy density of the vacuum** receives an infinite contribution from the **zero-point energies** of the various modes of oscillation. For a bosonic scalar field:

$$H_b = \sum_p (a_p^\dagger a_p + \frac{1}{2}) \epsilon_p$$



$$\langle 0 | H_b | 0 \rangle = \frac{1}{2} \sum_p \epsilon_p$$

Fermionic  $s=1/2$  fields give a negative contribution:

$$H_f = \sum_p (b_p^\dagger b_p + c_p^\dagger c_p - 1) \epsilon_p$$



$$\langle 0 | H_f | 0 \rangle = - \sum_p \epsilon_p$$

- The scale of the zero-point energy density is provided by the cutoff:

$$\rho_{\text{vac}} = \frac{1}{V} \langle 0 | H | 0 \rangle \sim \frac{1}{V} \sum_{\varepsilon_p < \Lambda_{\text{cut}}} \varepsilon_p \stackrel{(\varepsilon_p = cp)}{\approx} \Lambda_{\text{cut}}^4 / (\hbar c)^3$$

**If**  $\Lambda_{\text{cut}} \sim M_{\text{Planck}}$  ➔  $\rho_{\text{vac}} \sim 10^{123} \rho_{\text{vac}}^{\text{obs}}$

Exact SUSY would solve the problem:

$$\langle 0 | H | 0 \rangle = \left( \frac{1}{2} n_b - n_f \right) \sum_p \varepsilon_p = 0$$

But SUSY is broken  
Assuming  $\Lambda_{\text{SUSY}} \approx 1 \text{ TeV}$ :

$$\rho_{\text{vac}} \approx \Lambda_{\text{SUSY}}^4 / (\hbar c)^3 \sim 10^{59} \rho_{\text{vac}}^{\text{obs}}$$

The problem of the scale of the cosmological constant has found no solution so far

Modification of gravity (extra-dim), leak of vacuum energy to other universes (wormholes),...

# Baryogenesis (matter-antimatter asymm.)

- So far, no primordial antimatter has been observed in the Universe. Up to distances of order 0.1-1 Gpc the Universe consists only of matter. [1Gpc =  $3.2 \cdot 10^9$  light years. Observable universe:  $H_0 \sim 10$  Gpc ]
- A very **plausible assumption** is that the big bang produces an equal number of quarks and antiquarks

## WHEN AND WHY ANTIMATTER DISAPPEARED ?

The Sakharov conditions: (1967)

- 1) Baryon number violation
- 2) C and CP violation
- 3) Departure from thermal equilibrium

In the SM:

- Istanton process
- Weak interactions
- EW phase transition

In the SM, for  $m_H \geq 80$  GeV, the e.w. phase transition does not provide enough thermal instability necessary for baryogenesis

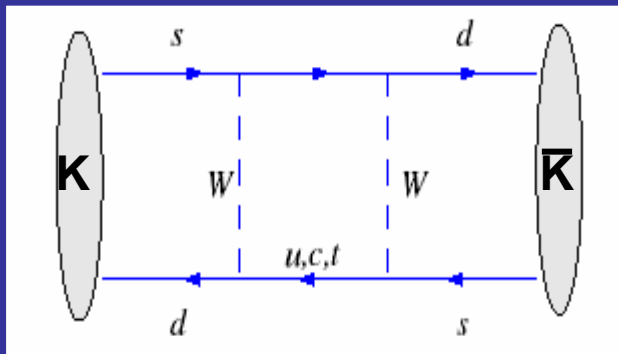
$\mathcal{CP}$  generated by the CKM mechanism is irrelevant for baryogenesis  
→ Non-standard  $\mathcal{CP}$  is a necessary ingredient for baryogenesis

# FLAVOUR PHYSICS AS A PROBE OF NEW PHYSICS

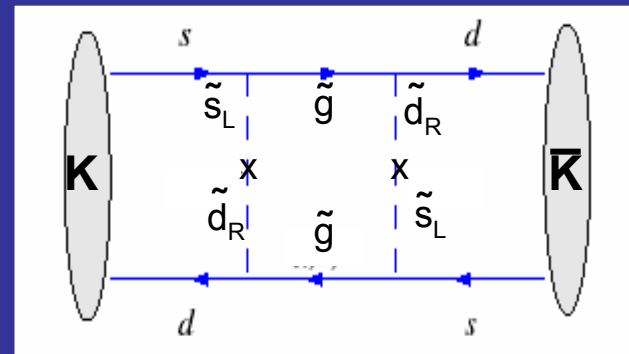
# INDIRECT SEARCHES OF NEW PHYSICS

Most of **beyond SM physics** can be explained by New Physics models (**SUSY, GUT, Extra-dim., ...**).

New Physics enters the low-energy processes through quantum loops  $\longrightarrow$  **Flavor Physics could allow us to discriminate among different New Physics scenarios**



Standard Model



New Physics

In the past, **the existence of the charm quark or the heaviness of the top quark** have been predicted from the study of virtual effects long before their experimental observation



# THE STANDARD MODEL AS AN EFFECTIVE LOW-ENERGY THEORY

The great success of the SM up to energies of  $O(\Lambda_{EW})$  scale tells us that **the SM** has to be recovered as **the low-energy limit of the New Physics theory**:

$$\mathcal{L}_{eff} = O(\Lambda^2) \mathcal{L}_2 + O(\Lambda) \mathcal{L}_3 + O(1) \mathcal{L}_4 + \\ + O(1/\Lambda) \mathcal{L}_5 + O(1/\Lambda^2) \mathcal{L}_6 + \dots$$

**Standard Model:**  
Renormalizable  
operators

**New Physics:**  
suppressed by  
 $O(1/\Lambda^{4-i})$

$\mathcal{L}_2$ : Boson masses  $\varphi^2$ . In the SM  $\delta m_H^2 \sim O(\Lambda^2) \rightarrow$  Hierarchy problem

$\mathcal{L}_3$ : Fermion masses  $\bar{\psi}\psi$ . Protected by chiral symmetry:  $\Lambda \rightarrow m \log \Lambda$

$\mathcal{L}_4$ : Renormalizable interactions:  $\bar{\psi}\gamma A\psi$

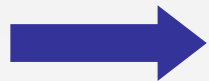
$\mathcal{L}_5$ : Neutrino masses:  $\nu^T \nu H^2, \dots$   $\mathcal{L}_6$ :  $(\bar{\psi}\gamma\psi)(\bar{\psi}\gamma\psi), \dots$

**Which is the scale of New Physics?**

# THE "NATURAL" NEW PHYSICS SCALE

$$m_H^2 = m_{\text{bare}}^2 + \delta m_H^2$$
$$\delta m_H^2 = \frac{3G_F}{\sqrt{2}\pi^2} m_t^2 \Lambda^2 + \dots \approx (0.3 \Lambda)^2$$

But radiative  
corrections indicate  
 $m_H \sim 100 \text{ GeV}$



The "natural" cut-off is  $\Lambda = O(1 \text{ TeV})$

New Physics must be very "special":  
it is so close, but its effects are not directly visible

The SM works too well both in  
the **electroweak** and in the **flavour** sector



**THE LEP PARADOX**



**THE FLAVOR PROBLEM**

# THE FLAVOUR PROBLEM

Consider the most general effective Hamiltonian for  $\Delta F = 2$  processes:

$$\langle M | H_{\text{eff}}^{\text{NP}} | \bar{M} \rangle = \sum_{i,j} C_j(\Lambda) W_{ji}(\Lambda, \mu) \underbrace{\langle M | Q_i(\mu) | \bar{M} \rangle}_{\text{Complete basis of four-fermion operators}}$$

NEW PHYSICS SCALE  $\uparrow$

Complete basis of four-fermion operators

The  $C(\Lambda)$  depend on the general properties of the NP model:

$$C_i(\Lambda) = a F_i / \Lambda^2$$

$a$  = loop factor  
 $F$  = flavour coupling

$$C_i(\Lambda) = a / \Lambda^2$$

Generic flavour structure

$$C_1(\Lambda) = a F_{\text{SM}} / \Lambda^2, \quad C_j(\Lambda) = 0$$

Minimal Flavour Violation

$$C_i(\Lambda) = a |F_{\text{SM}}| e^{i\varphi} / \Lambda^2$$

Next-to-Minimal Flavour Violation

$F_{\text{SM}}$  is the combination of CKM factors for the considered process

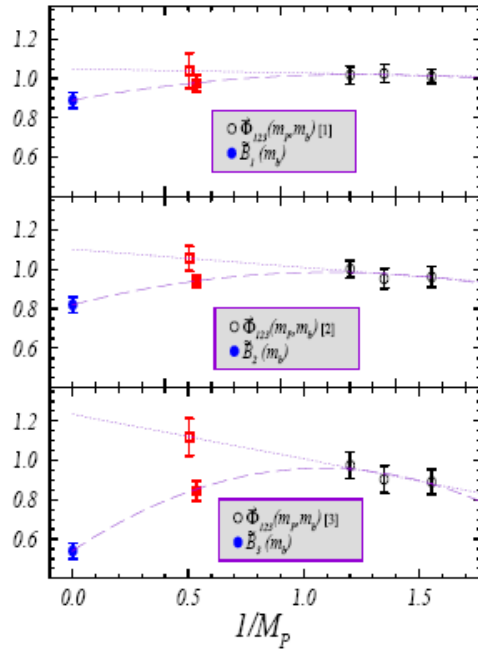
$a \sim 1$  for tree-level or strongly interacting NP

$a \sim \alpha_s$  or  $\alpha_w$  loop-mediated strong or weak interacting contributions

# Hadronic matrix elements from Lattice QCD

B- $\bar{B}$  MIXING

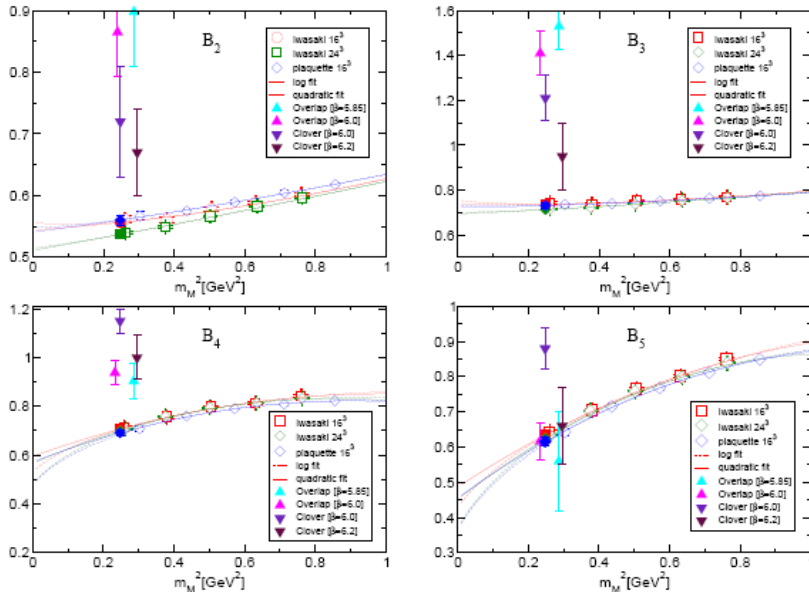
Becirevic et al.,  
2001

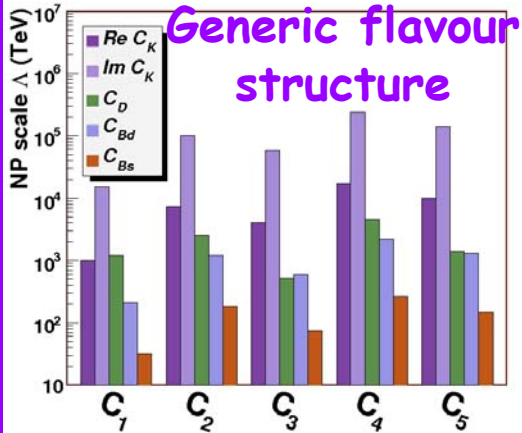


K- $\bar{K}$  MIXING

- ▼ ▼ APE 1999
- ▲ ▲ Babich et al. 2006
- ◆ ■ CP-PACS 2006

Only few calculations !!





## Lower bound on $\Lambda$ in TeV

Scenario	strong/tree	$\alpha_s$ -loop	$\alpha_w$ -loop
MFV	5	0.5	0.2
NMFV	62	6.2	2
<b>General</b>	<b>24000</b>	<b>2400</b>	<b>800</b>

**THE "FLAVOUR PROBLEM"**

NP contributions, particularly chirality flipping (LR) operators, are enhanced by the values of the hadronic matrix elements and by the RGE.

When these operators are allowed, particularly in the kaon sector, the NP scale is easily pushed beyond the TeV scale (and the LHC reach).

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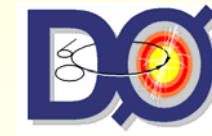
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evidence of New Physics in  $B_s$  mixing
- Lattice QCD and quark masses



# In the era of precision experimental flavour physics

$\epsilon_K$	$(2.280 \pm 0.013) 10^{-3}$	0.6%
$\Delta m_d$	$(0.507 \pm 0.005) \text{ ps}^{-1}$	1%
$\Delta m_s$	$(17.77 \pm 0.12) \text{ ps}^{-1}$	0.7%
$\text{Sin}2\beta$	$0.668 \pm 0.028$	4%
$ V_{us}  f_+(0)$	$0.21664 \pm 0.00048$	0.2%
...	...	...

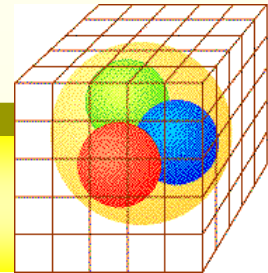


...

... ..

we are also entering the era of

## Precision LATTICE QCD



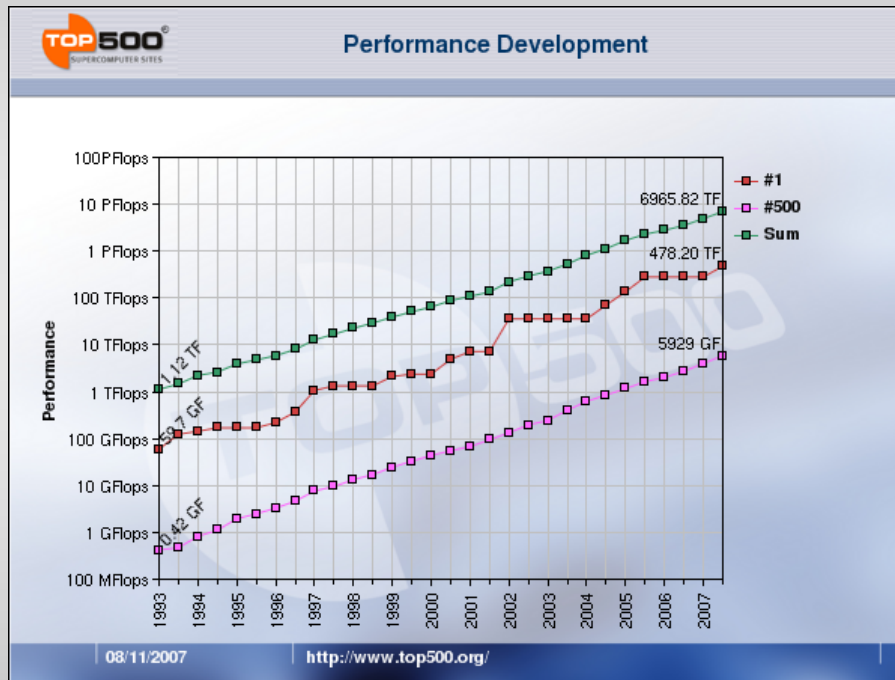
Unquenched calculations with relatively low quark masses are now being performed by several groups using different approaches (lattice action, renormalization,...).

Crucial when aiming at a percent precision.

# "PRECISION" LATTICE QCD: WHY NOW

1) Increasing of computational power  
 → Unquenched simulations

The Moore's Law



For Lattice QCD today:  
 ~ 1-30 TFlops

TeraFlops machines are required to perform unquenched simulations

CPU cost for  $N_f=2$  Wilson fermions:

[Del Debbio et al. 2006]

$$\text{TFlops-years} \approx 0.15 \left( \frac{N_{\text{conf}}}{100} \right) \left( \frac{L_s}{3 \text{ fm}} \right)^5 \left( \frac{L_t}{2L_s} \right) \left( \frac{0.15}{\hat{m} / m_s} \right) \left( \frac{0.08 \text{ fm}}{a} \right)^6$$



## 2) Algorithmic improvements: → Light quark masses in the ChPT regime

“The Berlin wall has been disrupted”

CPU cost (for  $N_f=2$  Wilson fermions):

Ukawa 2001 (The Berlin wall):

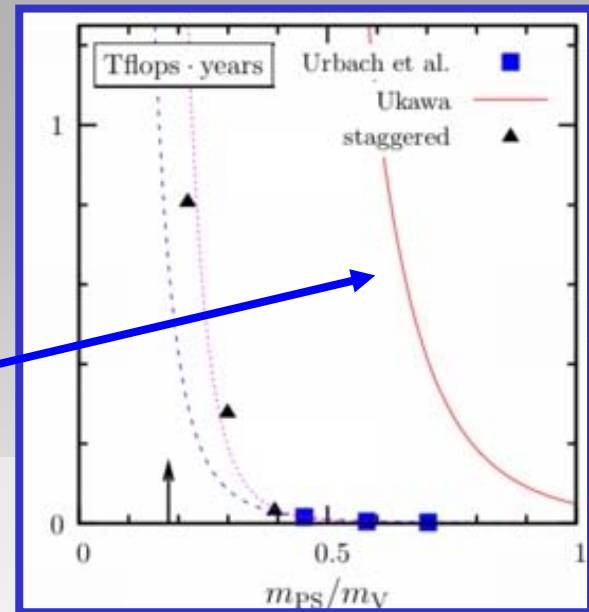
$$\text{TFlops-years} \approx 3.1 \left( \frac{N_{\text{conf}}}{100} \right) \left( \frac{L_s}{3 \text{ fm}} \right)^5 \left( \frac{L_t}{2L_s} \right) \left( \frac{0.2}{\hat{m} / m_s} \right)^3 \left( \frac{0.1 \text{ fm}}{a} \right)^7$$

Del Debbio et al. 2006:

$$\text{TFlops-years} \approx 0.03 \left( \frac{N_{\text{conf}}}{100} \right) \left( \frac{L_s}{3 \text{ fm}} \right)^5 \left( \frac{L_t}{2L_s} \right) \left( \frac{0.2}{\hat{m} / m_s} \right) \left( \frac{0.1 \text{ fm}}{a} \right)^6$$

Today:  $M_\pi^{\text{latt}} \approx 250 - 300 \text{ MeV}$  ( $\hat{m}_{ud}^{\text{latt}} / m_s \approx 1/6 - 1/8$ ) **ChPT**

Few years ago:  $M_\pi^{\text{latt}} \approx 500 \text{ MeV}$  ( $\hat{m}_{ud}^{\text{latt}} / m_s \approx 1/2$ )



Vus AND THE  
"FIRST ROW"  
UNITARITY TEST

# 1<sup>st</sup> row: the most stringent unitarity test

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Source: Nuclear  $\beta$ -dec. K13,K12       $b \rightarrow u$  semil.

Abs. error:  $5 \cdot 10^{-4}$        $5 \cdot 10^{-4}$        $\sim 10^{-6}$

The **PDG 2004** quoted a  $2\sigma$  deviation from unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.0029 \pm 0.0015$$

**Extraordinary experimental progress:** the old PDG average for  $V_{us}$  has been superseded by the new results:

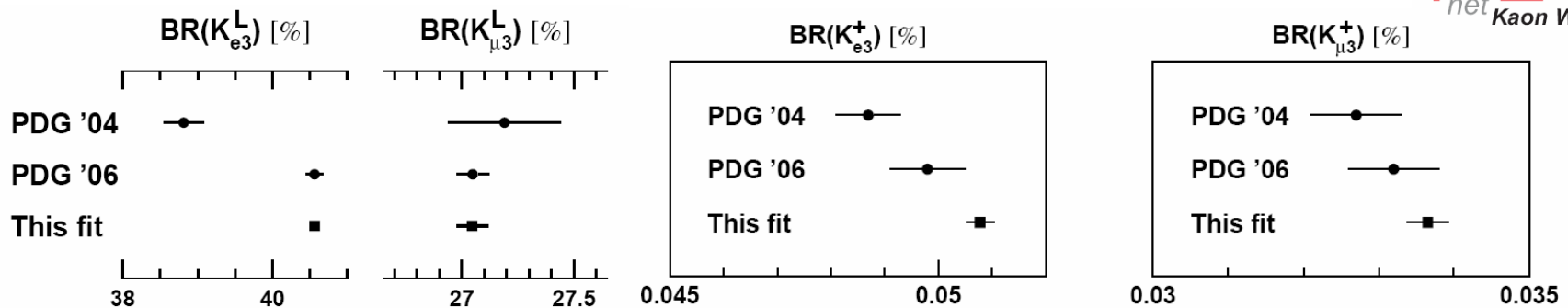
KLOE

ISTRA+

NA48

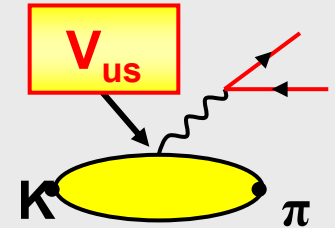
KTeV

FlaviA  
net  
Kaon WG



# V<sub>us</sub> from K13 decays

$$\Gamma_{K \rightarrow \pi l \nu} = C_K^2 \frac{G_F^2 m_K^5}{192 \pi^2} / S_{EW} [1 + \Delta_{SU(2)} + 2\Delta_{EM}] \times |V_{us}|^2 |f_+^{K\pi}(0)|^2$$



Ademollo-Gatto:  $f_+(0) = 1 - O(m_s - m_u)^2 \leftarrow O(1\%)$ . But represents the largest theoret. uncertainty

## ChPT

$$f_+(0) = 1 + f_2 + f_4 + O(p^8)$$

Vector Current Conservation

$f_2 = -0.023$   
Independent of  $L_i$   
(Ademollo-Gatto)

**THE LARGEST  
UNCERTAINTY**

Old standard estimate:  
Leutwyler, Roos (1984)  
(QUARK MODEL)  
 $f_4 = -0.016 \pm 0.008$

# ChPT calculation of $f_4$ and model estimates

$$f_4 = \Delta_{\text{loops}}(\mu) - \frac{8}{F_\pi^4} [C_{12}(\mu) + C_{34}(\mu)] (M_K^2 - M_\pi^2)^2$$

Post, Schilcher, 2001; Bijmans and Talavera, 2003

$C_{12}(\mu)$  and  $C_{34}(\mu)$  can be determined (in principle) from the slope and the curvature of the scalar form factor. But experimental data are not accurate enough.

A model dependence must be introduced:

Leutwyler and Roos,	$f_4^{\text{LOC}} = -0.016 \pm 0.008$	[Quark model]
Jamin et al.,	$f_4^{\text{LOC}} = -0.018 \pm 0.009$	[Dispersive analysis]
Cirigliano et al.,	$f_4^{\text{LOC}} = -0.002 \pm 0.008$	[1/Nc+Low resonance]

# Lattice QCD

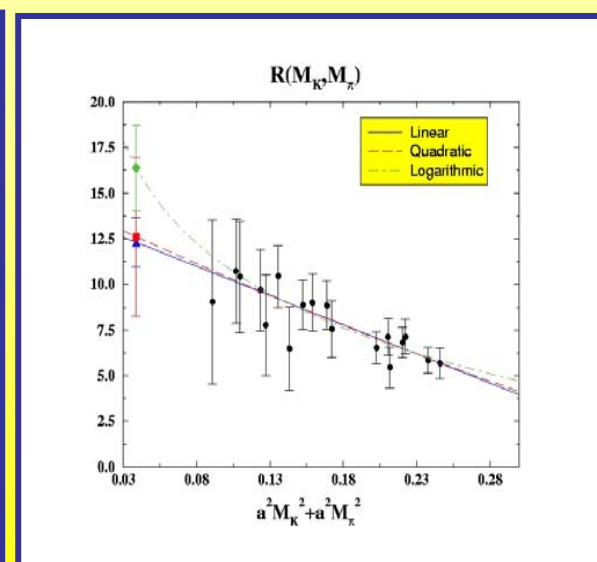
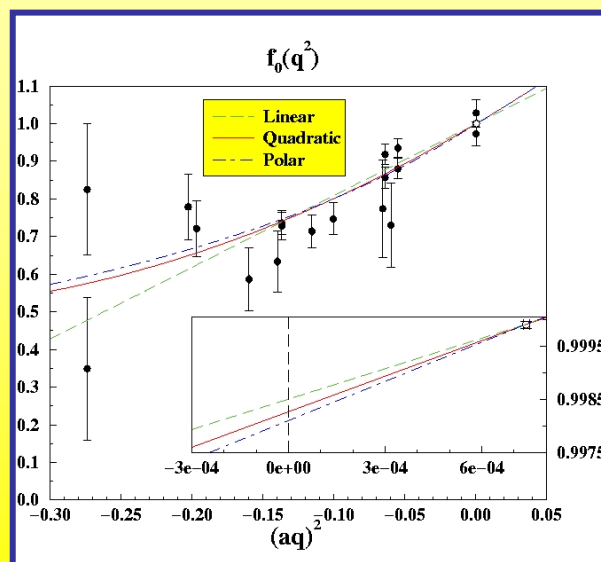
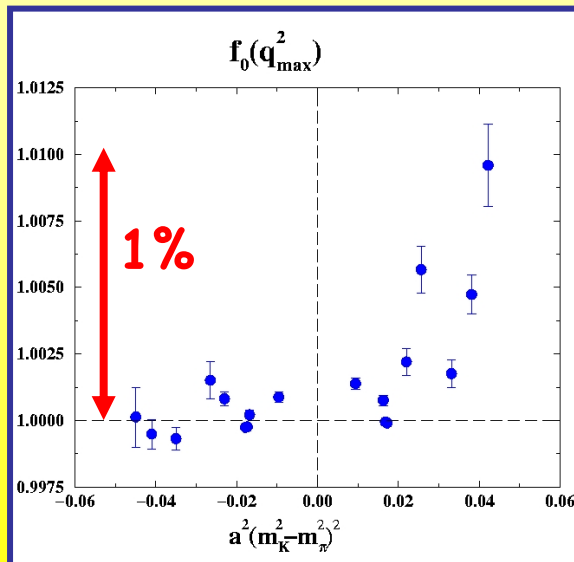
THE REQUIRED **O(1%)** PRECISION HAS BEEN REACHED

D.Becirevic, G.Isidori, V.L., G.Martinelli, F.Mescia, S.Simula, C.Tarantino, G.Villadoro. [NPB 705,339,2005]

The basic ingredient is a **double ratio** of correlation functions:

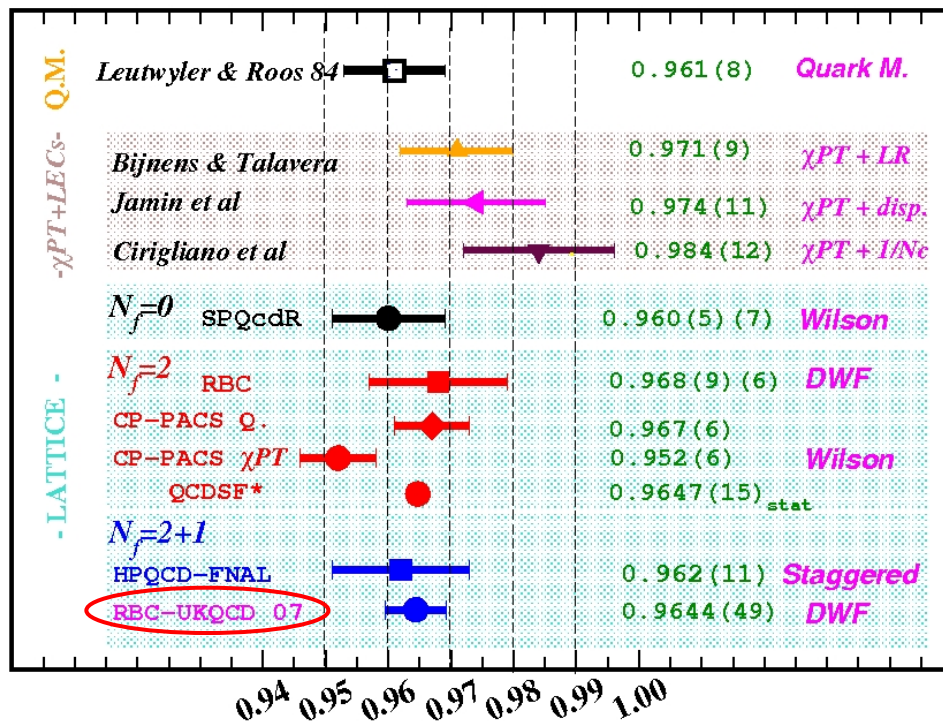
[FNAL for B→D/D\*]

$$R = \frac{\langle \pi | \bar{s} \gamma_0 u | K \rangle \langle K | \bar{u} \gamma_0 s | \pi \rangle}{\langle \pi | \bar{u} \gamma_0 u | \pi \rangle \langle K | \bar{s} \gamma_0 s | K \rangle} = \frac{(M_K + M_\pi)^2}{4M_K M_\pi} f_0(q_{max}^2)^2$$



# $f_+(0)$ : LATTICE SUMMARY

$$f_+^{K^0\pi^+}(0)$$



- Good agreement between  $N_f=2$  and  $2+1$  calculations and the first quenched result

- The error on  $\Delta f = f_+(0) - 1 - f_2$  quoted in the original calculation was 50%

- A new precise  $N_f=2+1$  calculation by RBC/UKQCD

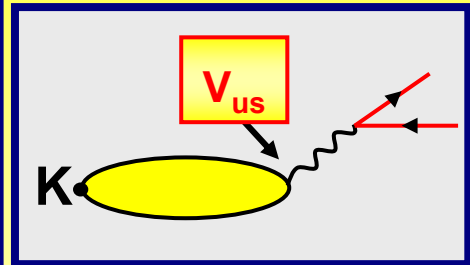
- Analytical (model dependent) results slightly higher than Lattice QCD

$$f_+(0) = 0.964(5) \rightarrow |V_{us}| = 0.2246(12)$$

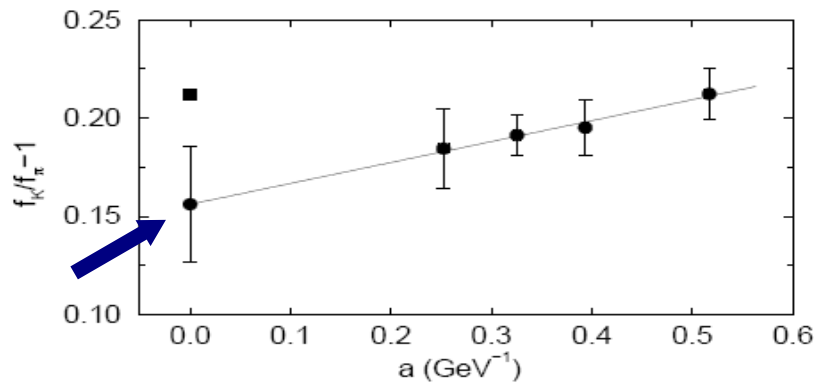
A. Jüttner@Latt'07 &  
Flavianet Kaon WG

# $V_{us}/V_{ud}$ from $K\mu 2/\pi\mu 2$ decays

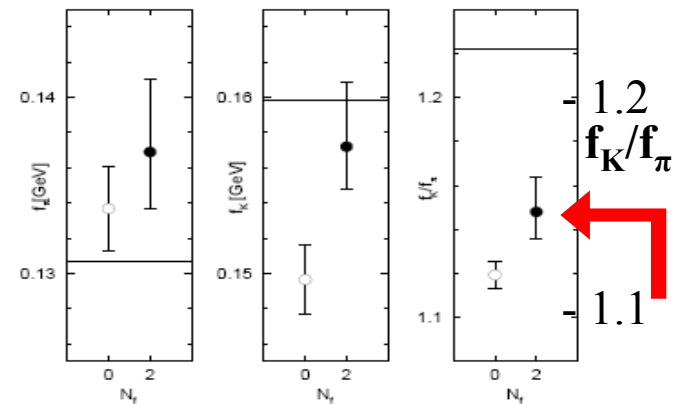
$$\frac{\Gamma(K \rightarrow \mu \bar{\nu}_\mu(\gamma))}{\Gamma(\pi \rightarrow \mu \bar{\nu}_\mu(\gamma))} = \frac{|V_{us}|^2 \left(\frac{f_K}{f_\pi}\right)^2 m_K \left(1 - \frac{m_\mu^2}{m_K^2}\right)}{|V_{ud}|^2 m_\pi \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)} \times 0.9930(35) \quad [\text{Marciano 04}]$$



**Nf=0 CP-PACS 2000**



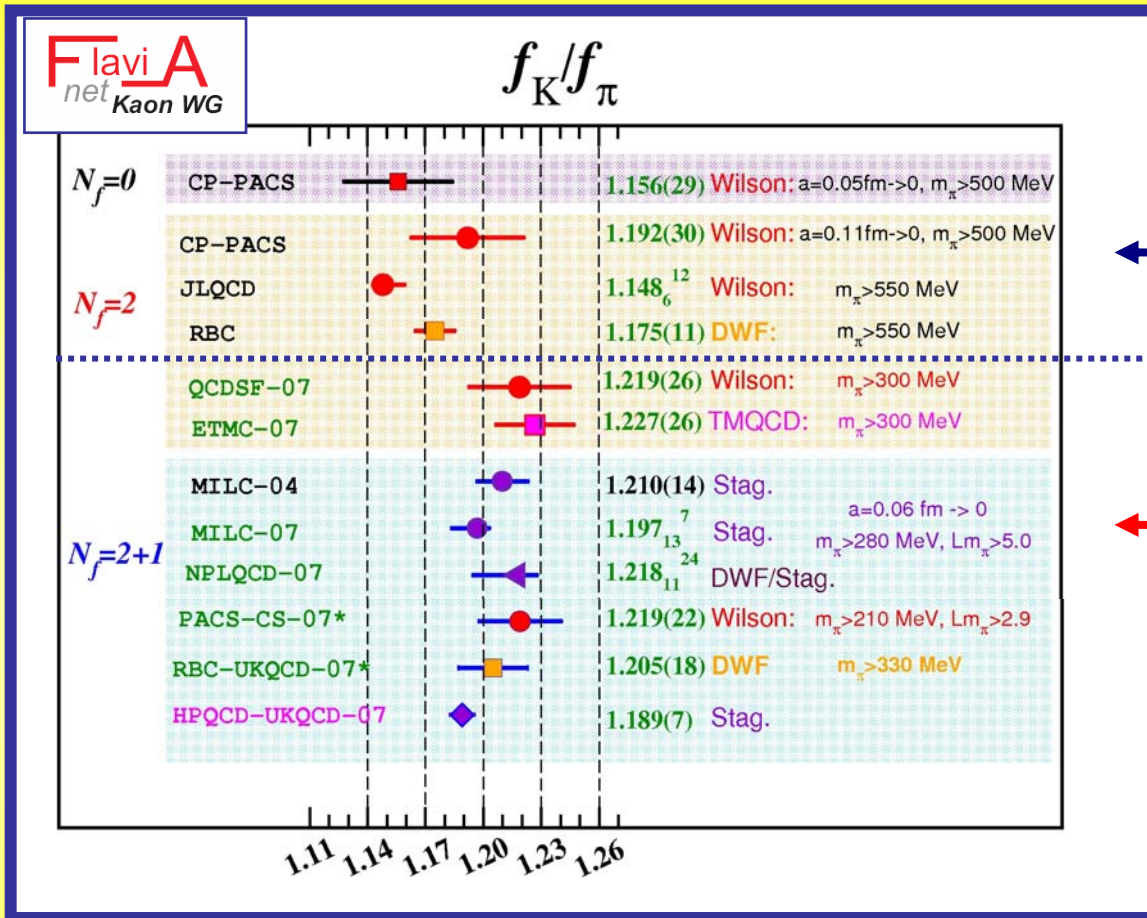
**Nf=0,2 JLQCD 2002**



- **Quenched calculations** typically indicated  $f_K/f_\pi - 1 \simeq 0.15$ , 25% smaller than the experimental value
- **Similar results** obtained by the **first unquenched calculations**
- A common feature of these calculations:  $M_\pi \gtrsim 500 \text{ MeV}$



# $f_K/f_\pi$ : LATTICE SUMMARY



“Heavy” quark masses

“Light” quark masses: many new results in the last year

$f_K/f_\pi = 1.198(10) \Rightarrow |V_{us}| = 0.2241(24)$

A.Jüttner@Latt'07

$f_K/f_\pi = 1.189(7) \Rightarrow |V_{us}| = 0.2261(15)$

Flavianet Kaon WG <sub>33</sub>



# LATTICE QCD AND THE UNITARITY TRIANGLE ANALYSIS

# THE UNITARITY TRIANGLES

Unitarity relations:

(Bjorken-Jarlskog)

$$V^\dagger V = 1 \longrightarrow \sum_k V_{ki}^* V_{kj} = \delta_{ij}$$

9 constraints,  
6 triangular relations

Only 2 triangles have all sides with length of the same  $O(\lambda^3)$ :

$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$

$$V_{ud} V_{td}^* + V_{us} V_{ts}^* + V_{ub} V_{tb}^* = 0$$

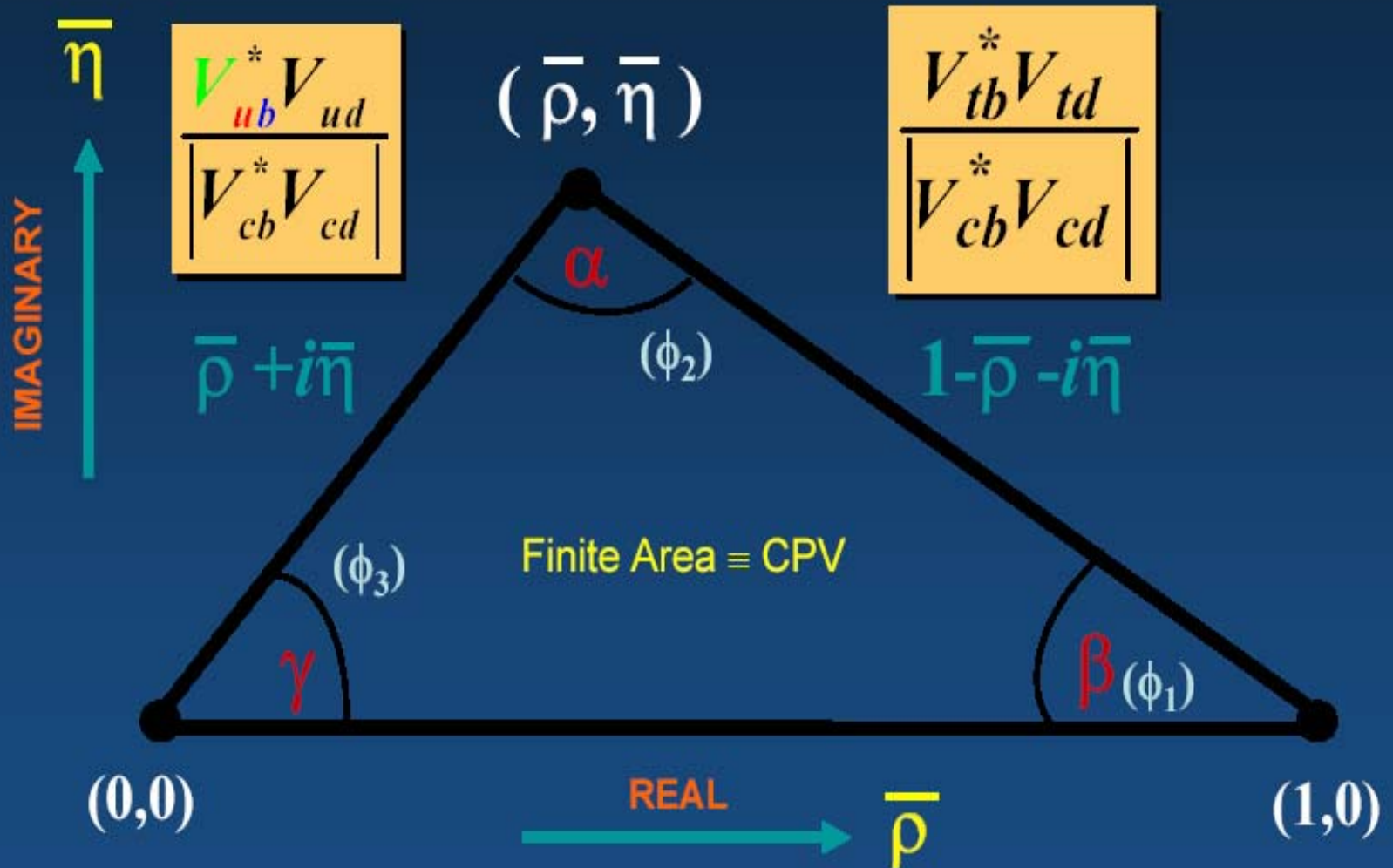
They are  
equivalent  
at order  $\lambda^3$

Only the orientation of the triangles depends on the phase convention. The **area** and  $\mathcal{CP}$  are proportional to:

$$J = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin \delta \approx A^2 \lambda^6 \eta \sim 10^{-5}$$

Unitarity:

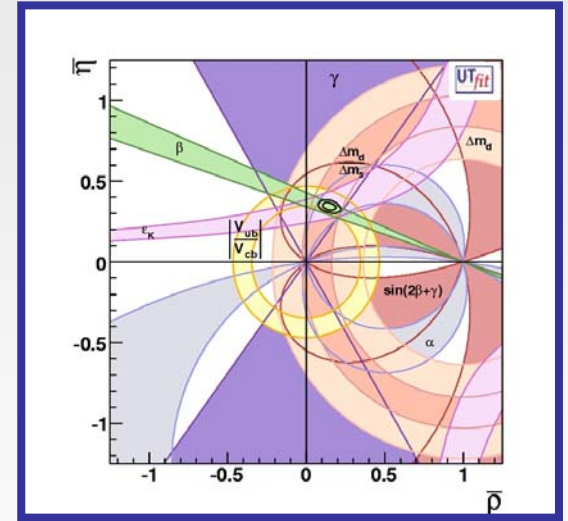
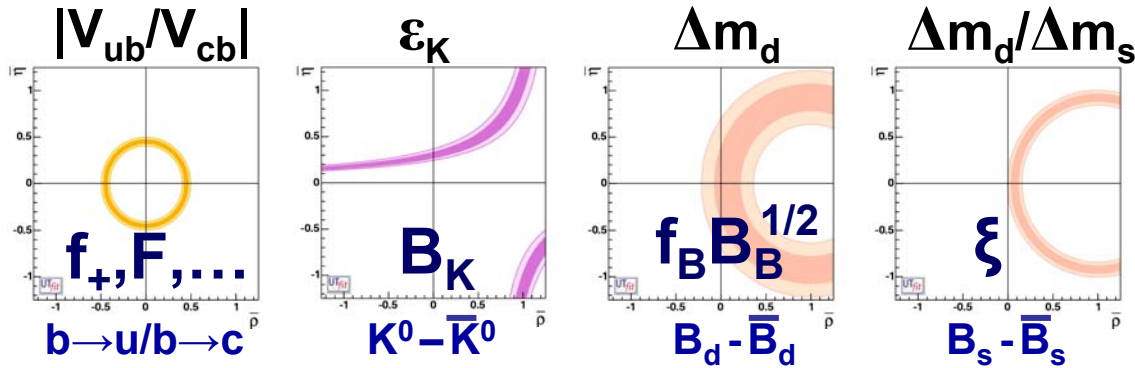
$$V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$$



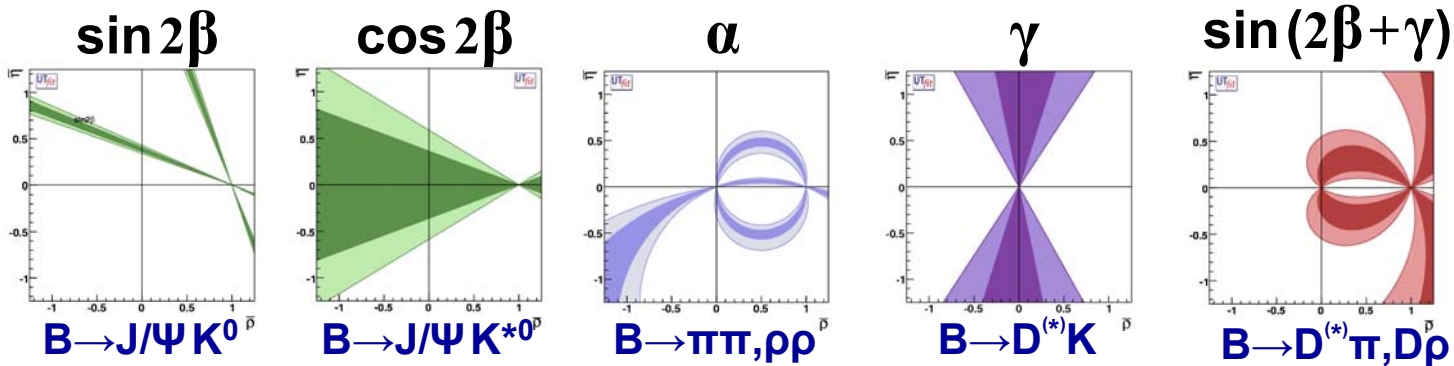
# THE UTA CONSTRAINTS



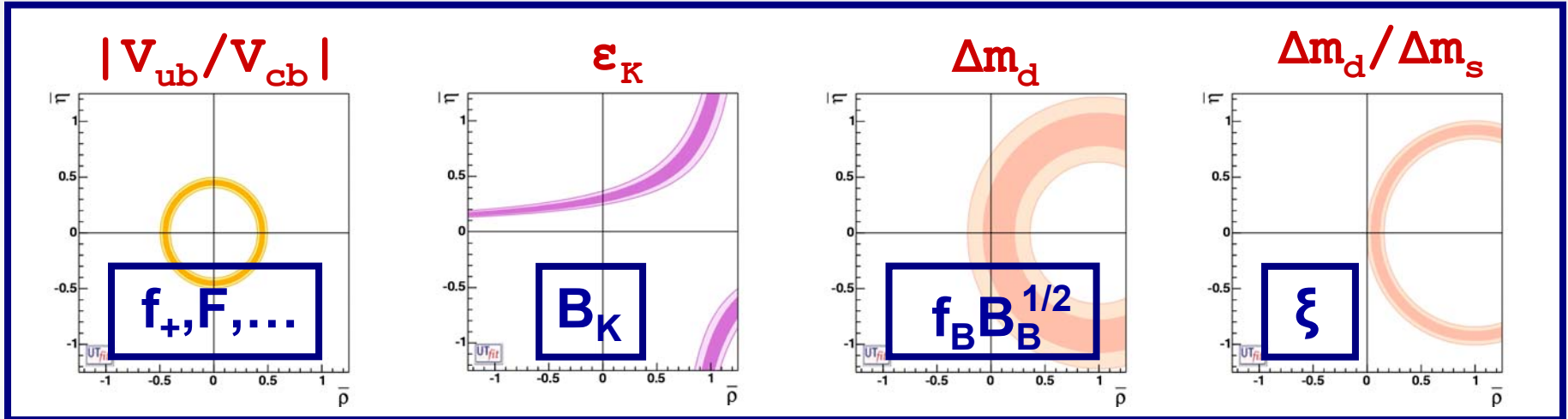
## UT-LATTICE



## UT-ANGLES



# THE "CLASSICAL" (pre-Bfactories) UT-LATTICE ANALYSIS



Hadronic matrix elements from **LATTICE QCD**

Already before the starting of the B factories

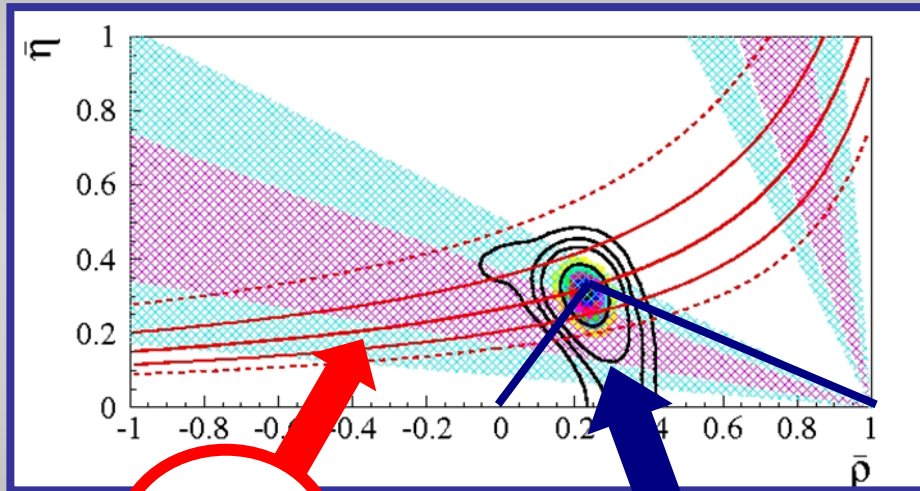
**3 IMPORTANT RESULTS FOR FLAVOUR PHYSICS**

A success of (quenched) **Lattice QCD** calculations



# 1) CKM EXPLANATION OF $\epsilon/\bar{p}$

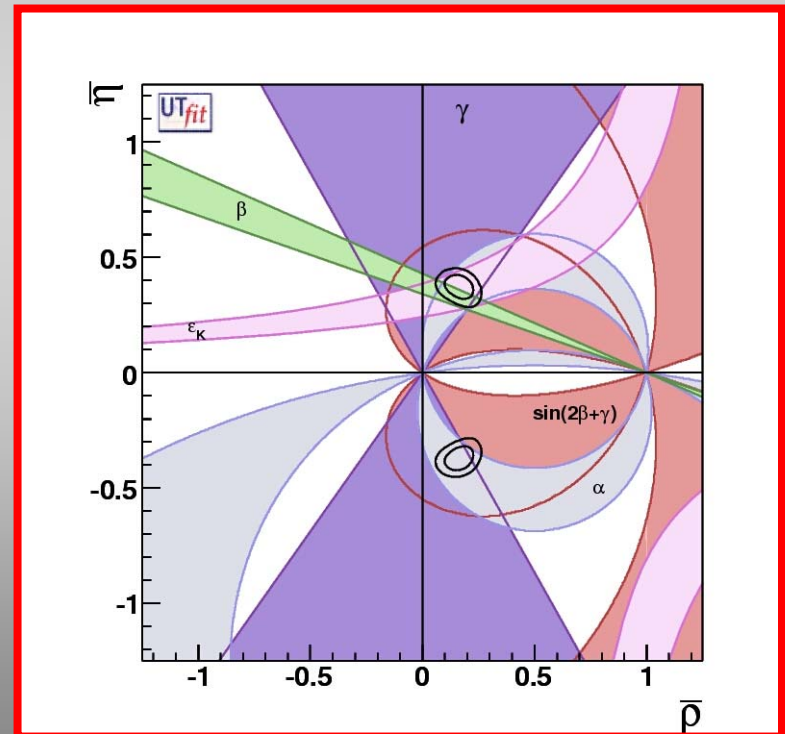
Ciuchini et al., 2000



$\epsilon_K$

UTsizes

UTfit, today





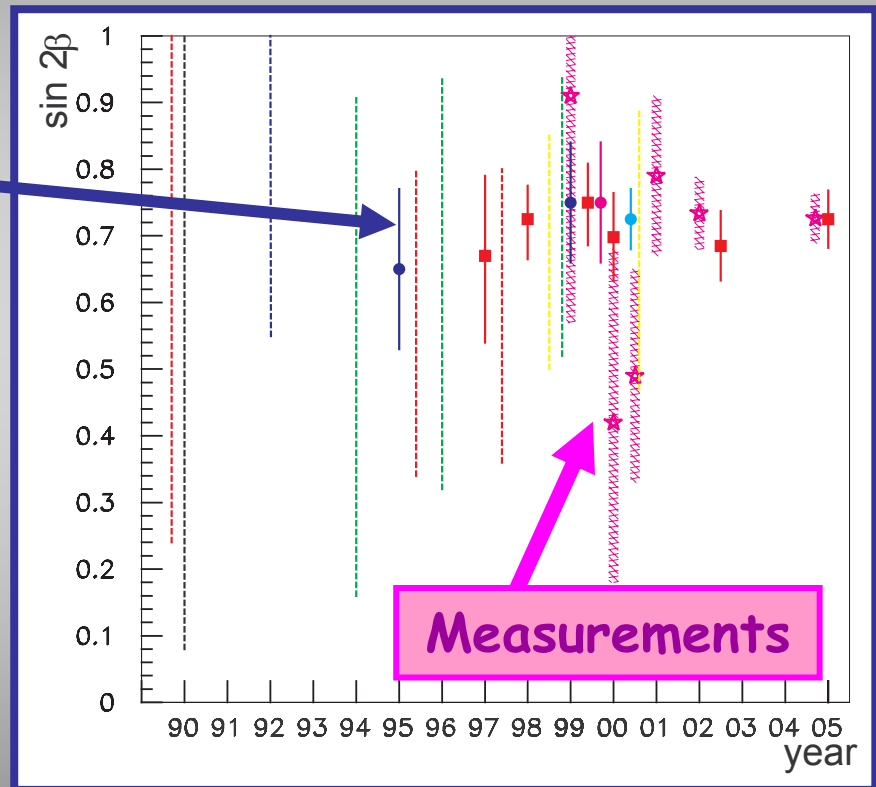
## 2) PREDICTION OF $\text{Sin}2\beta$

Predictions exist since 1995

Ciuchini et al., 1995:  
 $\text{Sin}2\beta_{\text{UTA}} = 0.65 \pm 0.12$

Ciuchini et al., 2000:  
 $\text{Sin}2\beta_{\text{UTA}} = 0.698 \pm 0.066$

UTfit today:  
 $\text{Sin}2\beta_{\text{UTA}} = 0.744 \pm 0.039$



Direct measurement today:  
 $\text{Sin}2\beta_{J/\psi K^*} = 0.681 \pm 0.025$

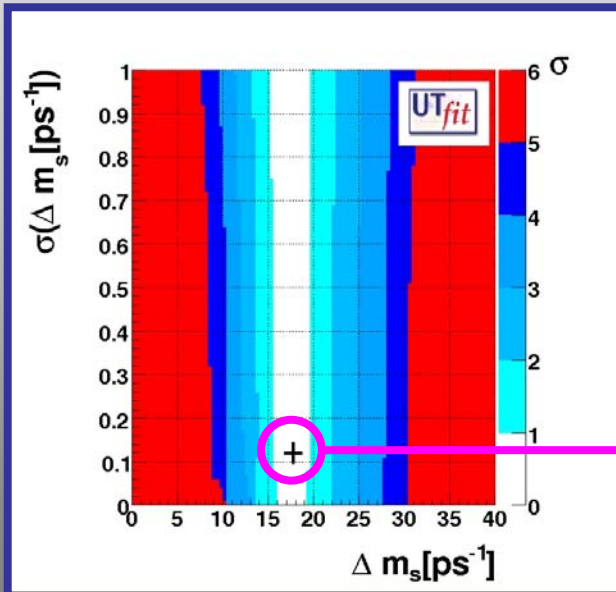
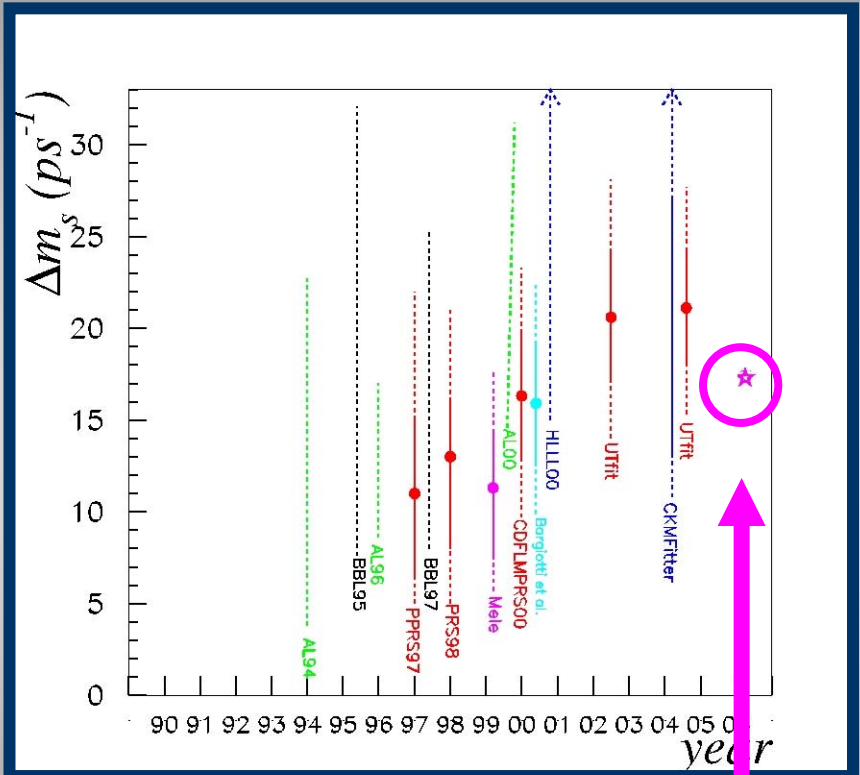
# 3) PREDICTION OF $\Delta m_s$

Ciuchini et al., 2000:

$$\Delta m_s = (16.3 \pm 3.4) \text{ ps}^{-1}$$

UTfit today:

$$\Delta m_s = (18.6 \pm 2.3) \text{ ps}^{-1}$$

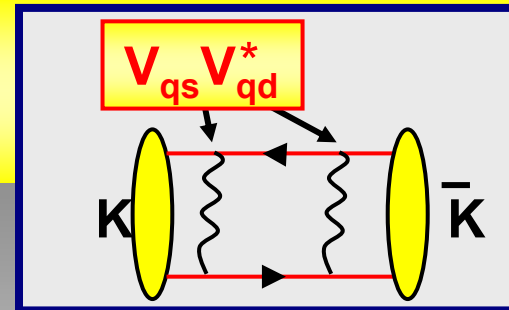
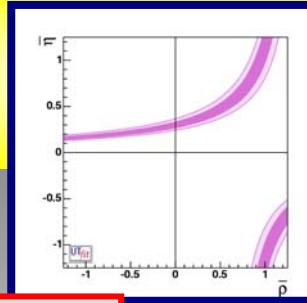


Direct measurement today

$$\Delta m_s = (17.77 \pm 0.12) \text{ ps}^{-1}$$

# $K^0 - \bar{K}^0$ mixing: $B_K$

$$\langle \bar{K}^0 | Q(\mu) | K^0 \rangle = \frac{8}{3} f_K^2 m_K^2 B_K(\mu)$$



$$\hat{B}_K = 0.86 \pm 0.05 \pm 0.14$$

L. Lellouch @ Latt'00

Quench.  
error

$$\hat{B}_K = 0.79 \pm 0.04 \pm 0.08$$

C. Dawson @ Latt'05

$$\hat{B}_K = 0.720 \pm 0.039$$

A. Jüttner @ Latt'07

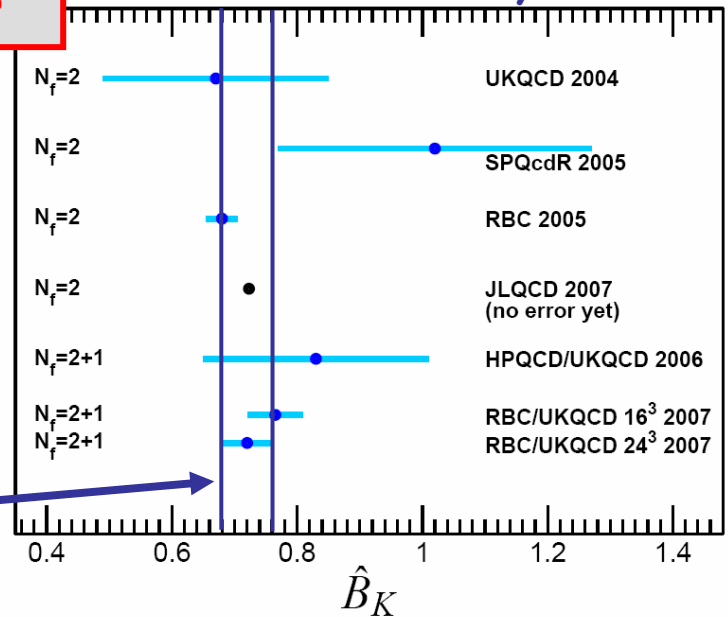
Precise results from chiral fermions

CP-PACS, 0803.2569 [hep-lat]

A very precise quenched calculation

$$\hat{B}_K = 0.782 \pm 0.005 \pm 0.007$$

A. Jüttner, Latt'07



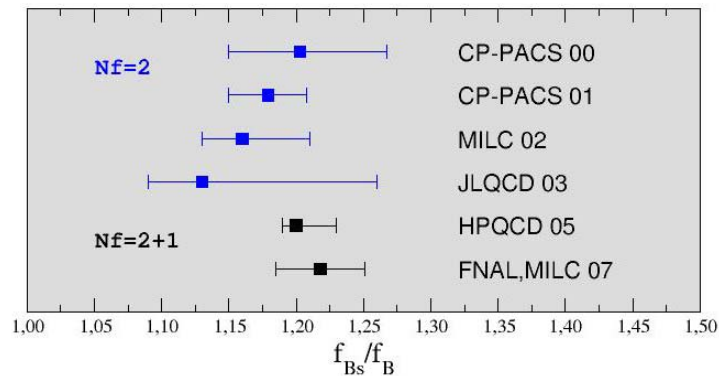
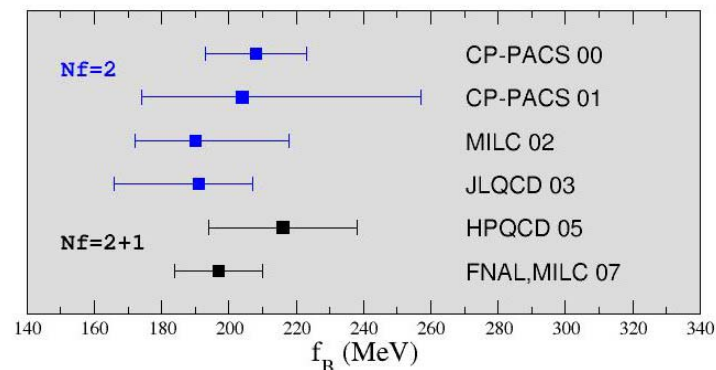
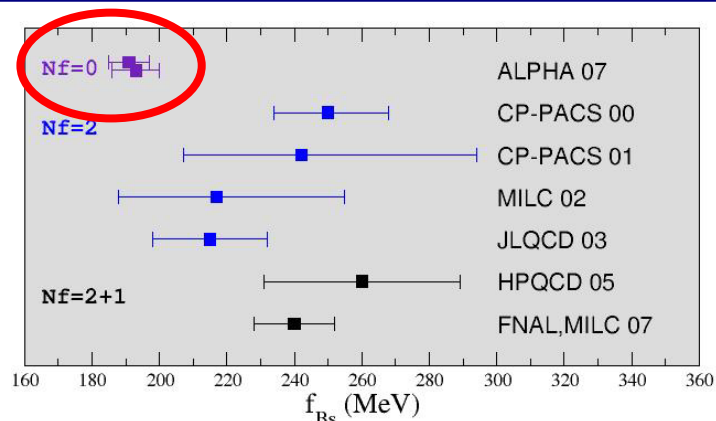
LQCD, Gavela et al., 1987:

$$\hat{B}_K = 0.90 \pm 0.20$$

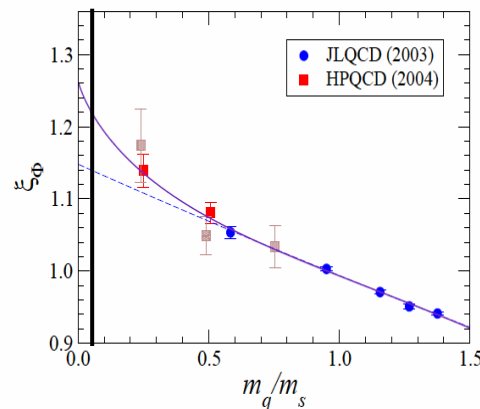
QCD SR, Pich, De Rafael, 1985:

$$\hat{B}_K = 0.33 \pm 0.09$$

# B-mesons decay constants: $f_B, f_{B_s}$



Inputs for  $\Delta m_{d/s}$  and  $B \rightarrow \tau \nu$



Chiral logs effects

Light  $m_q$  ( $< m_s/2$ ) crucial for  $f_{B_s}/f_B$

Kronfeld and Ryan, 2002:

$$f_{B_s}/f_B = 1.32 \pm 0.10$$

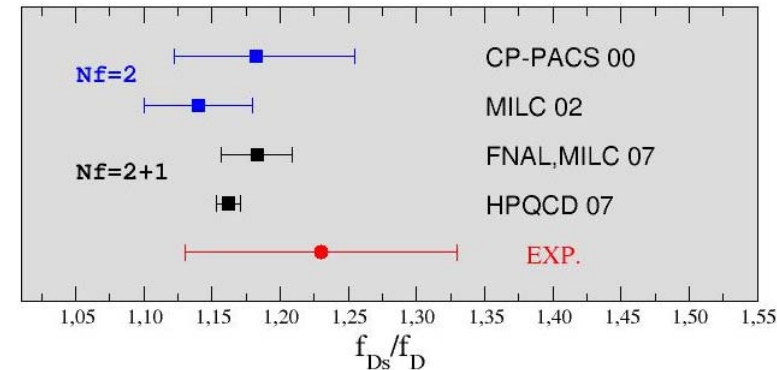
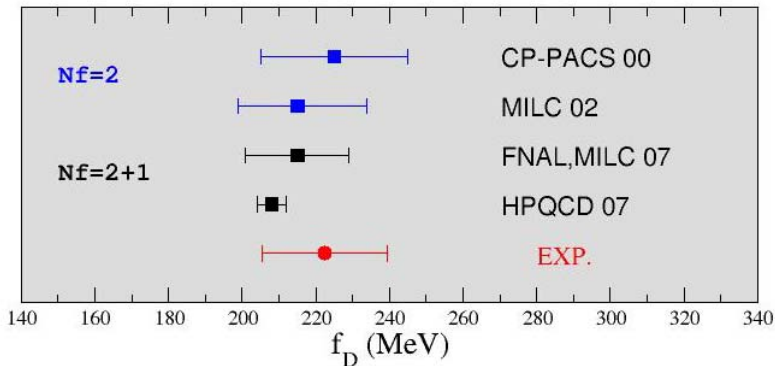
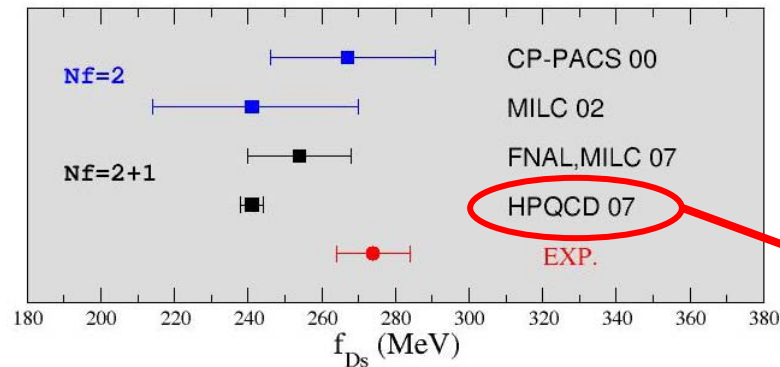
$$f_{B_s} = 230 \pm 30 \text{ MeV}$$

$$f_B = 189 \pm 27 \text{ MeV}$$

$$f_{B_s}/f_B = 1.23 \pm 0.06$$

Averages used in the UT fit

# D-mesons decay constants: $f_D, f_{D_s}$



“CLEO-c has the potential to provide a unique and crucial validation of LQCD”

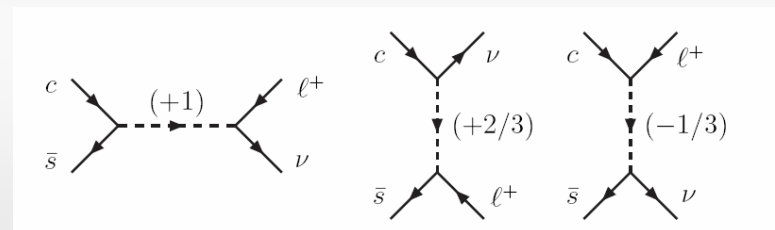
Ian Shipsey @ FPCP 2002

Co-Spokesperson of the CLEO Collaboration

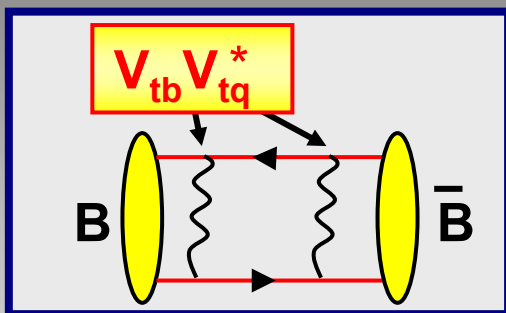
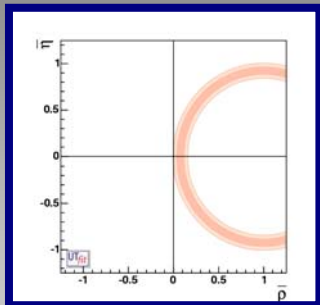
A new result by HPQCD, which claims a 1.2% precision on  $f_{D_s}$ , shows a discrepancy of about 3.5-4.0  $\sigma$  with the experimental average.

B. Dobrescu, A. Kronfeld, 0803.0512:

“Evidence for nonstandard leptonic decays of  $D_s$  mesons”



# B- $\bar{B}$ mixing: $B_{Bd}$ and $B_{Bs}$

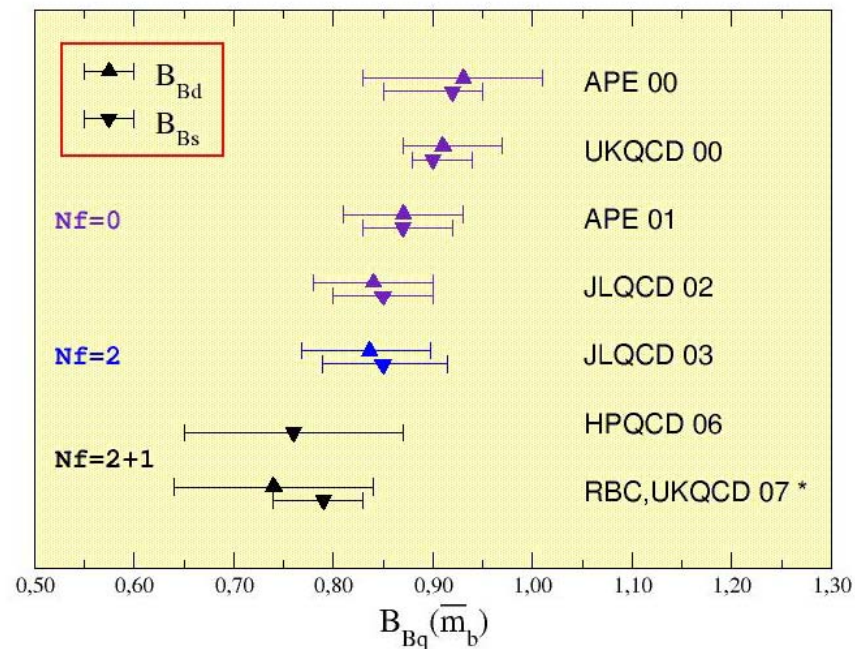


$$\langle \bar{B} | Q(\mu) | B \rangle = \frac{8}{3} m_B^2 f_B^2 B_B(\mu)$$

-Small chiral logs effects:

$$B_{Bd} \approx B_{Bs}$$

-Small quenching effects:  
consistent  $N_f=0$ ,  $N_f=2$   
and  $N_f=2+1$  results



Averages used in the UT fit

$$f_{Bs} \sqrt{\hat{B}_{Bs}} = 262 \pm 35 \text{ MeV}$$

$$\xi = 1.23 \pm 0.06$$

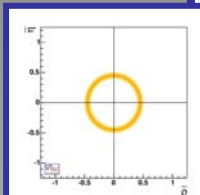
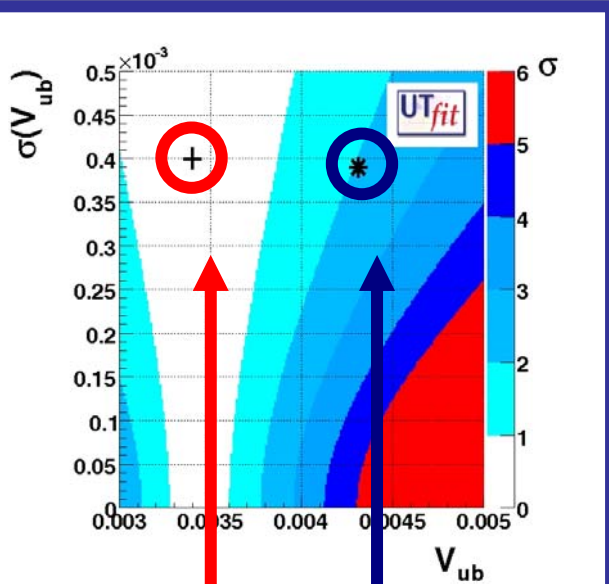
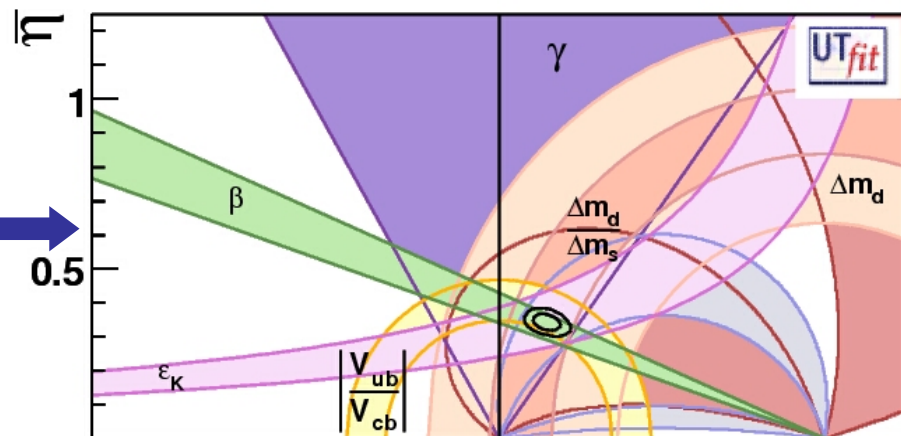
$$B_{Bd}(\bar{m}_b) = B_{Bs}(\bar{m}_b) =$$

$$0.84 \pm 0.03 \pm 0.06$$



# $V_{ub}$ from $B \rightarrow \pi l \nu$

Tension between  $V_{ub}$  inclusive and the UT fit



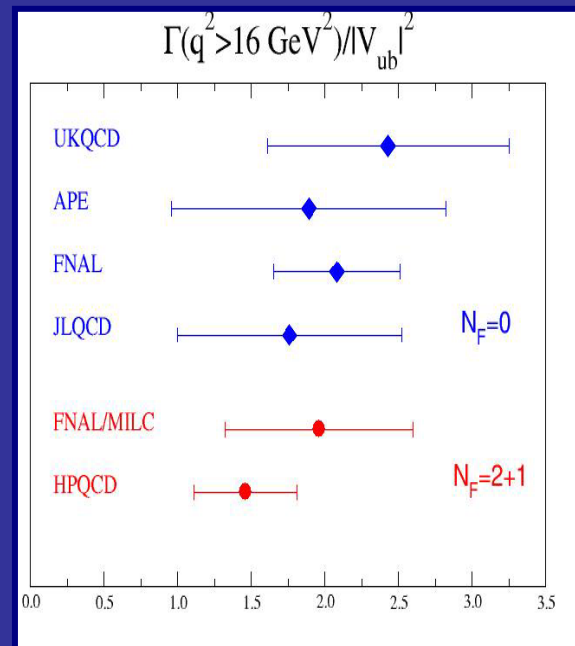
LATTICE QCD:  
Improve  $V_{ub}$   
exclusive to  
solve the tension

EXCLUSIVE:  $V_{ub}^{\text{excl.}} = (34.0 \pm 4.0) 10^{-4}$

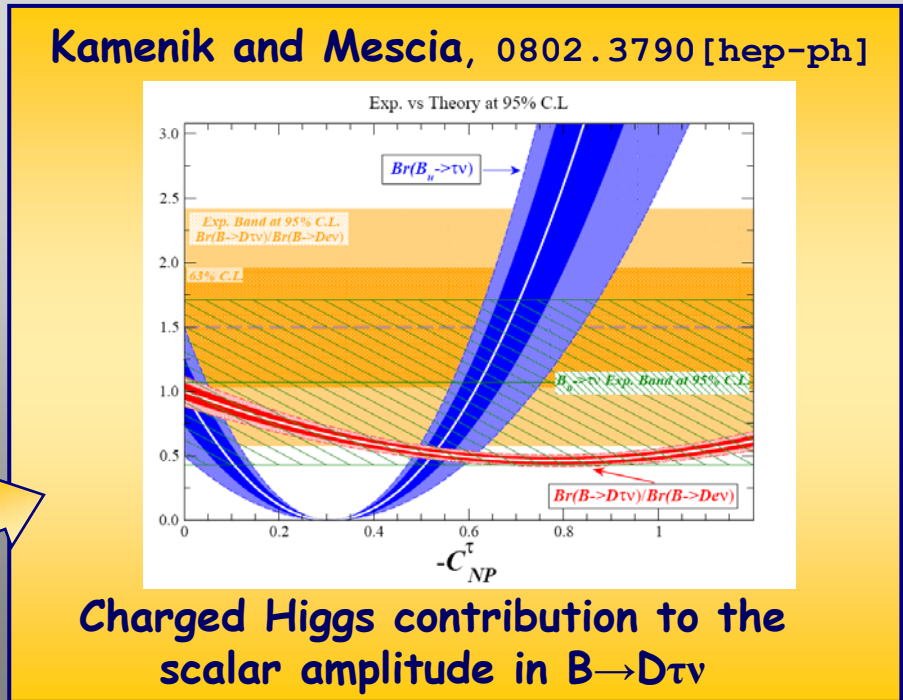
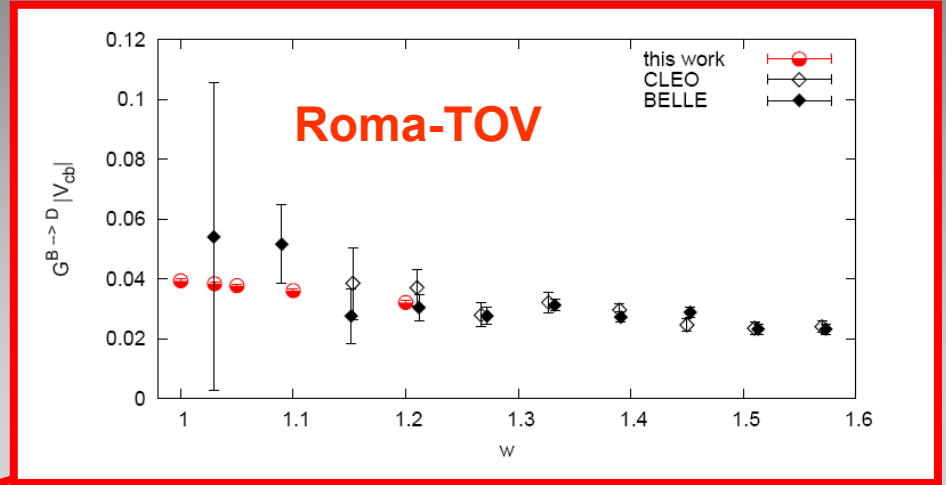
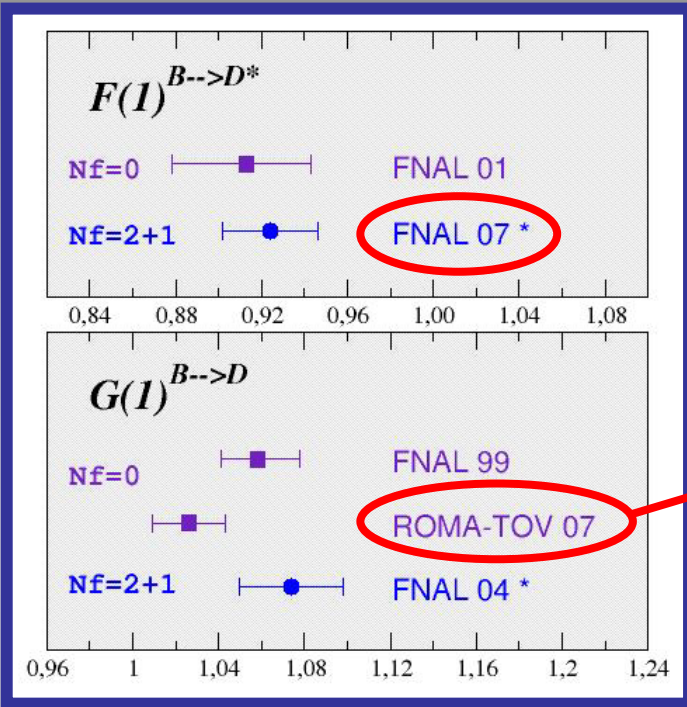
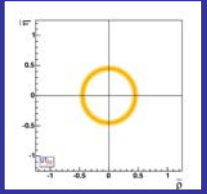
Form factors from LQCD and QCDSR

INCLUSIVE:  $V_{ub}^{\text{incl.}} = (43.1 \pm 3.9) 10^{-4}$

Model dependent in the threshold region  
(BLNP, DGE, BLL)



# Vcb from $B \rightarrow D/D^* l \nu$ decays



## Two new results

-FNAL:  $B \rightarrow D^*$ , Nf=2+1

-Roma TOV:  $B \rightarrow D$ , Nf=0:

New method (Step Scaling Method + Twisted b.c.),  $w \geq 1$ , both vector and scalar ff's.

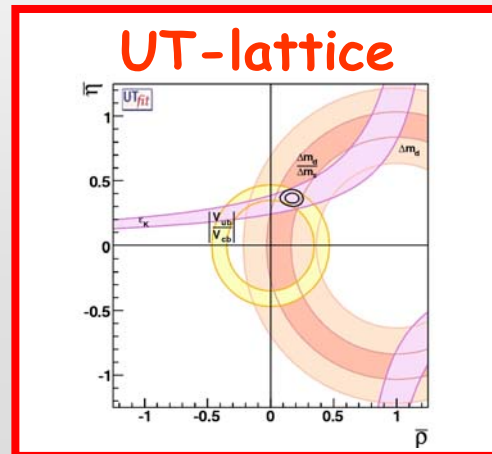
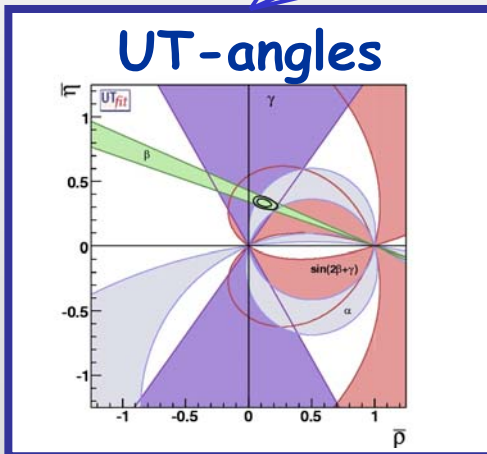


# "EXPERIMENTAL" DETERMINATION OF LATTICE PARAMETERS

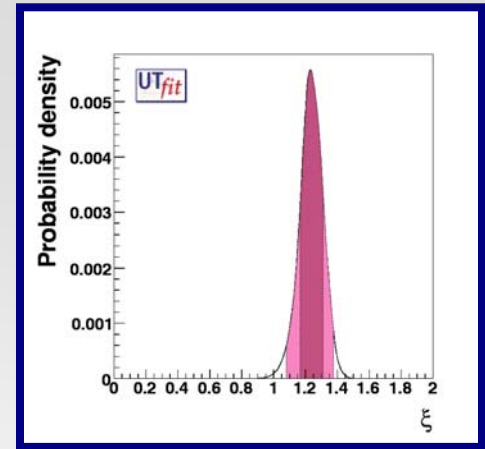
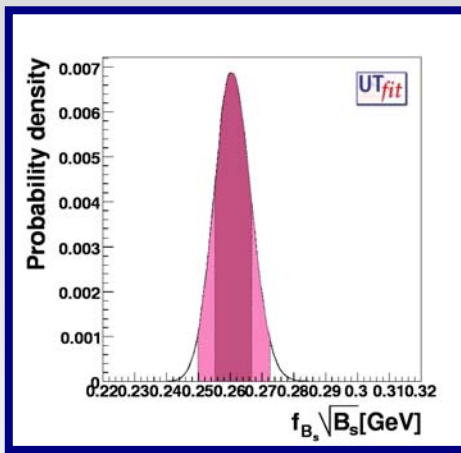
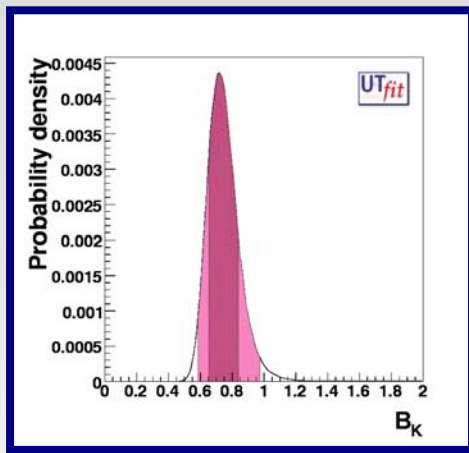
Assuming the validity of the Standard Model one can perform a simultaneous fit of the hadronic parameters:

$$|\varepsilon_K| = C_\varepsilon A^2 \lambda^6 \bar{\eta} \left[ -\eta_1 S(x_c) + \eta_2 S(x_t) \left( A^2 \lambda^4 (1 - \bar{\rho}) \right) + \eta_3 S(x_c, x_t) \right] \hat{B}_K$$

$$\Delta m_q = \frac{G_F^2}{6\pi^2} m_{B_q} M_W^2 \eta_B S_0(x_t) |V_{tq}|^2 \hat{B}_{B_q} f_{B_q}^2$$



Take the angles from experiments and extract  $f_{B_s} \sqrt{B_{B_s}}$ ,  $f_B \sqrt{B_{B_d}}$  or  $\xi$  and  $B_K$



**2%!** from  $\Delta m_s$

	$B_K$	$f_{B_s} \sqrt{B_{B_s}}$ (MeV)	$\xi$
<b>UTA</b>	<b><math>0.75 \pm 0.09</math></b>	<b><math>261 \pm 6</math></b>	<b><math>1.24 \pm 0.08</math></b>
<b>Lattice</b>	<b><math>0.79 \pm 0.04 \pm 0.08</math></b>	<b><math>262 \pm 35</math></b>	<b><math>1.23 \pm 0.06</math></b>

**Recent  
results**

**$0.720 \pm 0.039$**   
RBC/UKQCD 07

**$281 \pm 21$**   
HPQCD 06

**$1.21 \pm 0.03$**   
HPQCD, FNAL/MILC 07

Very good agreement. Increasing precision of LQCD.  
The role of LQCD in the SM UTA is not crucial...

... but Lattice QCD  
calculations are essential to  
perform

**THE UTA BEYOND THE  
STANDARD MODEL**

# THE UTA BEYOND THE SM: A MODEL INDEPENDENT ANALYSIS

New Physics in  $\Delta F = 2$  amplitudes can be parameterized in a simple general form. For instance:

$$C_{B_d} e^{2i\varphi_{B_d}} = \frac{\langle B_d^0 | H_{\text{eff}}^{\text{full}} | \bar{B}_d^0 \rangle}{\langle B_d^0 | H_{\text{eff}}^{\text{SM}} | \bar{B}_d^0 \rangle}$$

6 additional free parameters

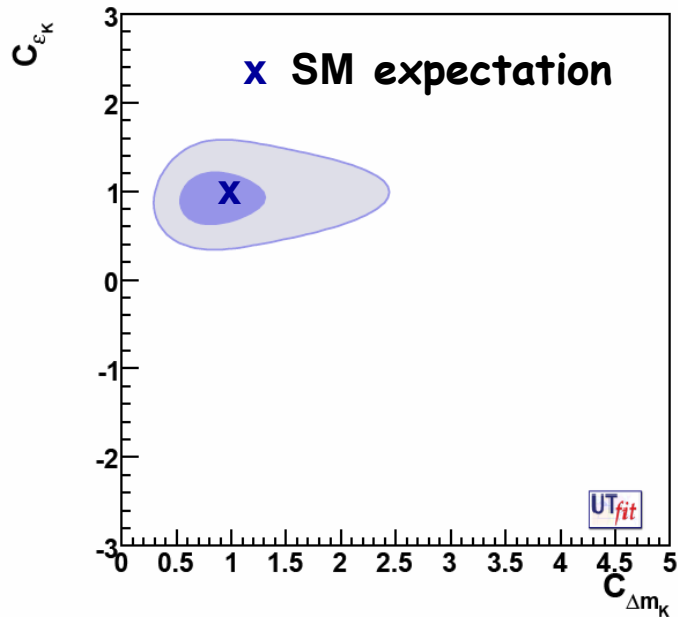
$$C_{B_d}, \varphi_{B_d}, C_{B_s}, \varphi_{B_s}, C_{\epsilon K}, C_{\Delta m K}$$

E.g.:  $(\Delta m_d)^{\text{exp}} = C_{B_d} (\Delta m_d)^{\text{SM}}$  ,  $\sin 2\beta^{\text{exp}} = \sin 2(\beta^{\text{SM}} + \varphi_{B_d})$

In the Standard Model:  $C_{xx} = 1$  ,  $\varphi_{xx} = 0$

A number of additional constraints are included:  
semileptonic asymmetries ( $A_{SL}^d, A_{SL}^s$ ), dimuon charge asymmetry ( $A_{SL}^{\mu\mu}$ ), lifetime differences and mixing phases ( $\Delta\Gamma_d/\Gamma_d, \Delta\Gamma_s/\Gamma_s, \phi_s$ )

## K-K mixing



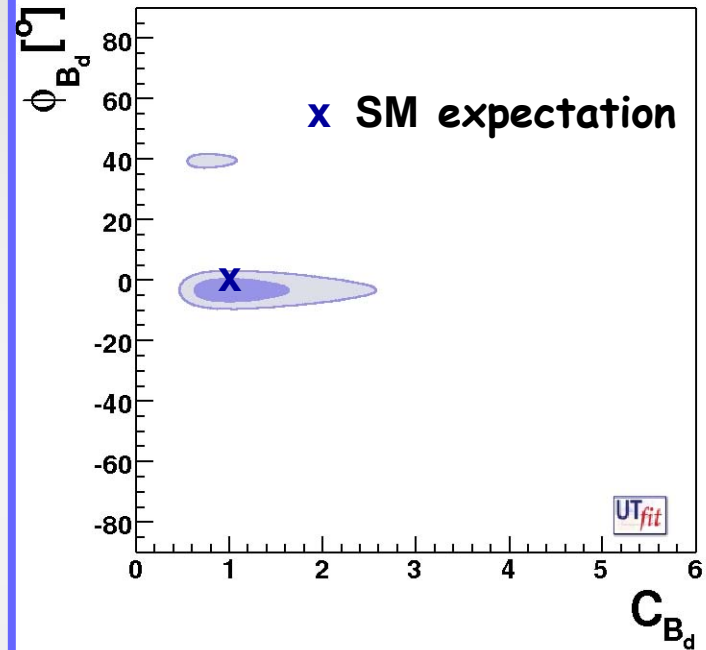
$$C_{\Delta m_K} = 0.93 \pm 0.32$$

$$C_{\epsilon_K} = 0.92 \pm 0.14$$



0707.0636  
[hep-ph]

## $B_d$ - $B_d$ mixing

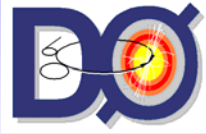


$$C_{B_d} = 1.05 \pm 0.34$$

$$\phi_{B_d} = (-3.4 \pm 2.2)^\circ$$

The  $V_{ub}$  tension produces a  
 $1.5\sigma$  effect in  $\phi_{B_d}$

# EVIDENCE OF NEW PHYSICS IN $B_s$ MIXING

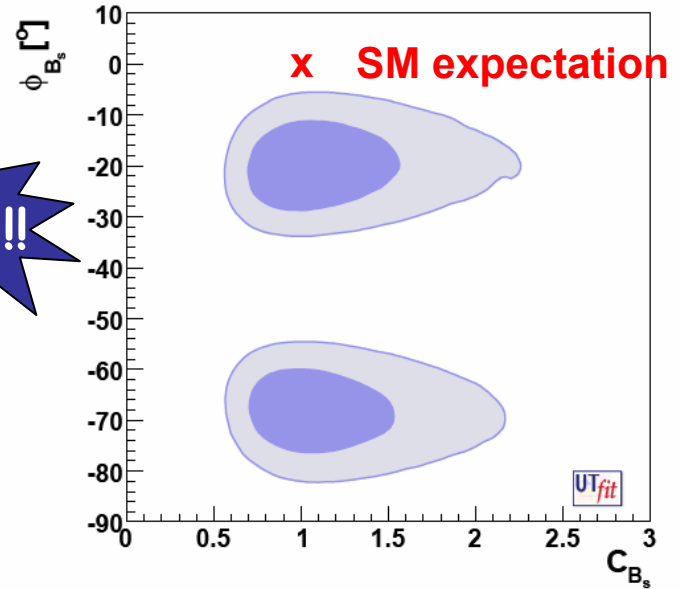
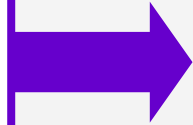


$\Delta\Gamma$  and  $\phi_{B_s}$   
from the

time-dependent angular  
analysis of  $B_s \rightarrow J/\psi \Phi$



0803.0659  
[hep-ph]



The accuracy on  $f_{B_s} \sqrt{B_{B_s}}$  required  
to observe NP in  $\Delta m_s$  at  $3\sigma$  is:

	1.7%	3.3%	5.0%
for $C_{B_s}$	1.1	1.2	1.3

The present accuracy is  $\sim 7-10\%$

$$C_{B_s} = 1.07 \pm 0.29$$

$$\phi_{B_s} = (-19.9 \pm 5.6)^\circ \cup (-68.2 \pm 4.9)^\circ$$

$$(\Delta m_s)^{\text{exp}} = C_{B_s} (\Delta m_s)^{\text{SM}}$$

$$\sin 2\beta_s^{\text{exp}} = \sin 2(\beta_s^{\text{SM}} + \phi_{B_s})$$

# LATTICE QCD AND QUARK MASSES

◆ **QUARK MASSES** CANNOT BE DIRECTLY MEASURED IN THE EXPERIMENTS, BECAUSE QUARKS ARE CONFINED INSIDE HADRONS

◆ BEING **FUNDAMENTAL PARAMETERS** OF THE STANDARD MODEL, **QUARK MASSES** CANNOT BE DETERMINED BY THEORETICAL CONSIDERATIONS ONLY.

➔ **QUARK MASSES** CAN BE DETERMINED BY COMBINING TOGETHER A **THEORETICAL** AND AN **EXPERIMENTAL** INPUT. E.G.:

$$[M_{\text{HAD}}(\Lambda_{\text{QCD}}, m_q)]^{\text{TH.}} = [M_{\text{HAD}}]^{\text{EXP.}}$$

**LATTICE QCD**



# LATTICE DETERMINATION OF QUARK MASSES

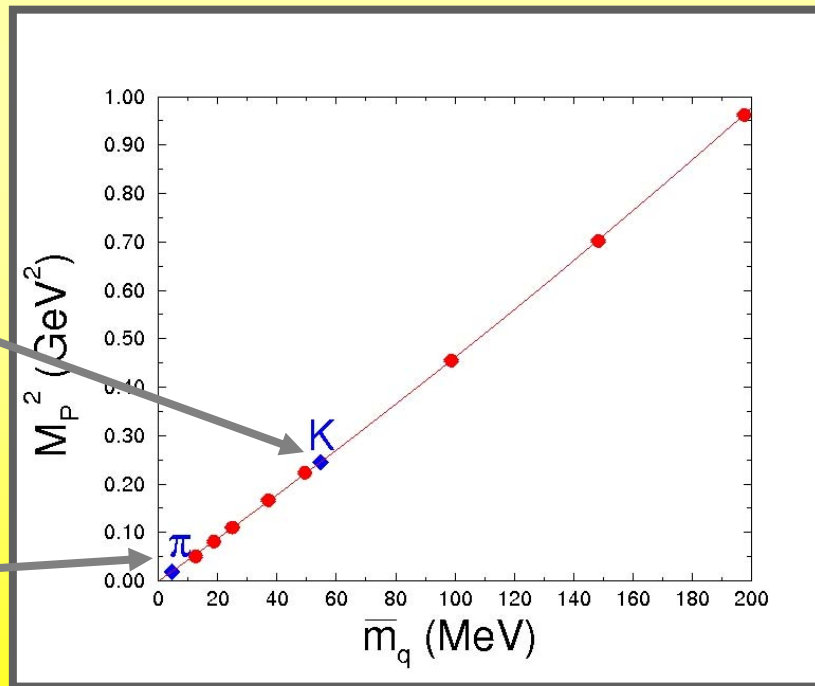
$$\hat{m}_q(\mu) = m_q(a) Z_m(\mu a)$$

ADJUSTED UNTIL  
 $M_H^{\text{LATT}} = M_H^{\text{EXP}}$

PERTURBATION THEORY OR  
NON-PERTURBATIVE METHODS

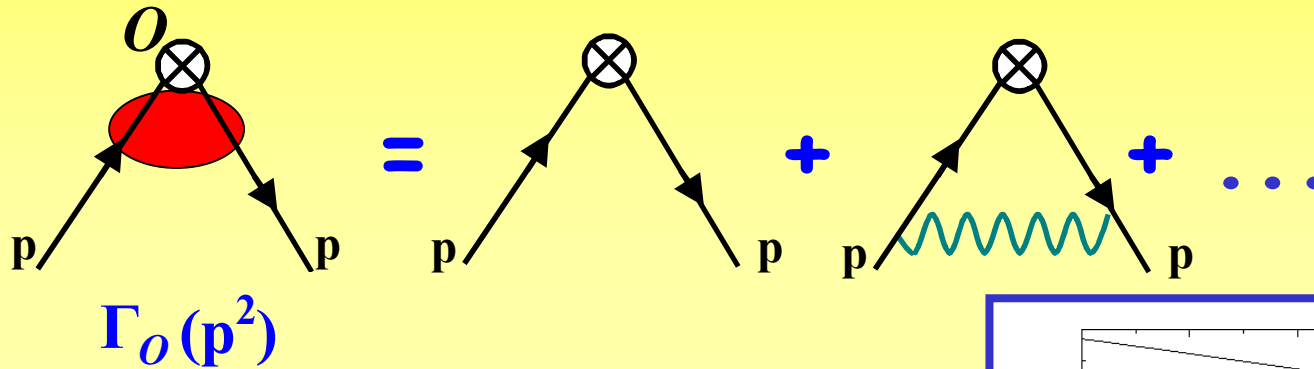
Extrapolation  
to  $m_q = m_s$

Extrapolation  
to  $m_q = m_{u,d}$



The effect of  
non-perturbative  
renormalization  
can be as large  
as 50%

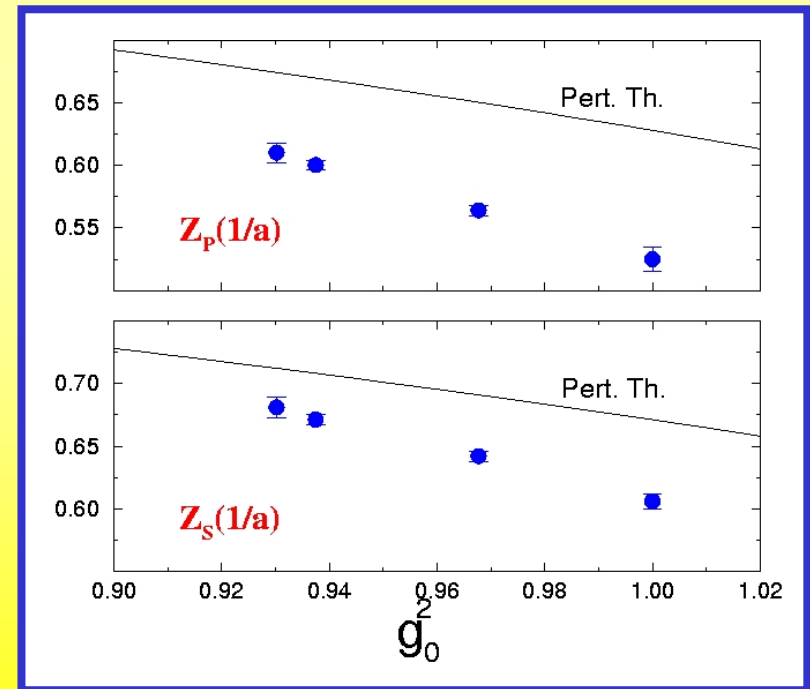
# NON-PERTURBATIVE RENORMALIZATION THE RI-MOM METHOD



The (non-perturbative)  
renormalization condition:

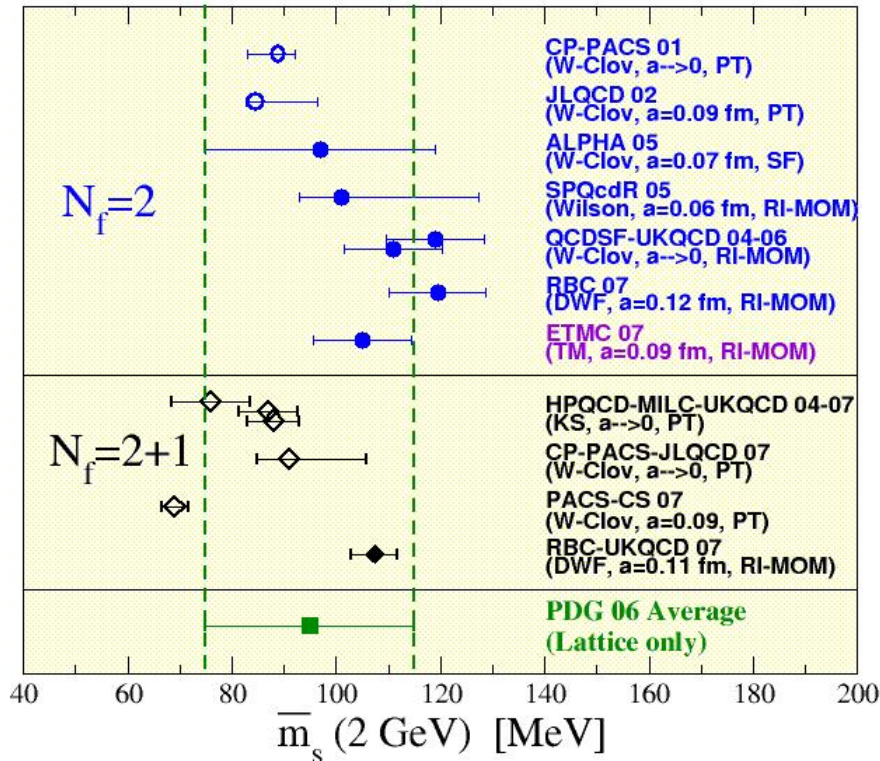
$$Z_O(a\mu) \Gamma_O(p^2)|_{p^2=\mu^2} = \Gamma_{\text{Tree-Level}}$$

Several NPR techniques have been developed: **Ward Identities**, **Schrodinger functional**, **X-space**



# ms: LATTICE SUMMARY

from ETM Collaboration, 0710.0329 [hep-lat]



(\* ) Empty symbols:  
perturbative  
renormalization

CP-PACS, 0803.2569 [hep-lat]

A very “precise” quenched  
calculation

$$m_s^{\text{MS}}(2 \text{ GeV}) = 105.6 (1.2) \text{ MeV}$$

The same accuracy can be  
reached in unquenched  
determinations

The error introduced by the use of perturbative renormalization is typically larger than other systematic effects, including quenching

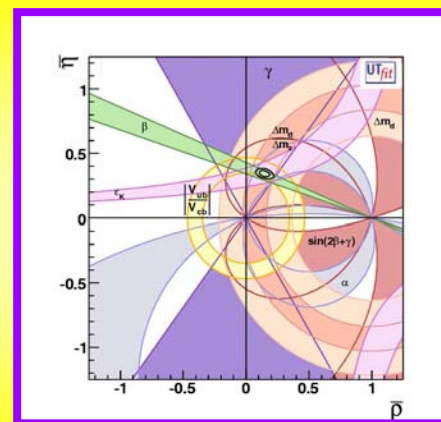
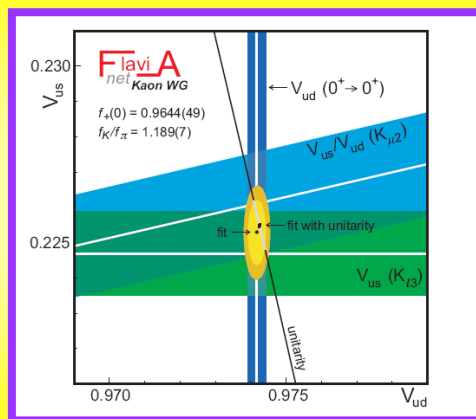
# LATTICE QCD AND FLAVOUR PHYSICS

The past

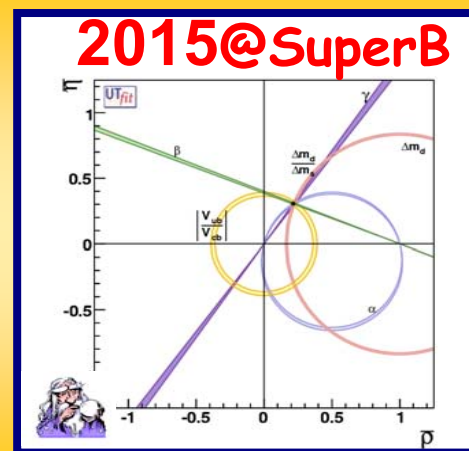
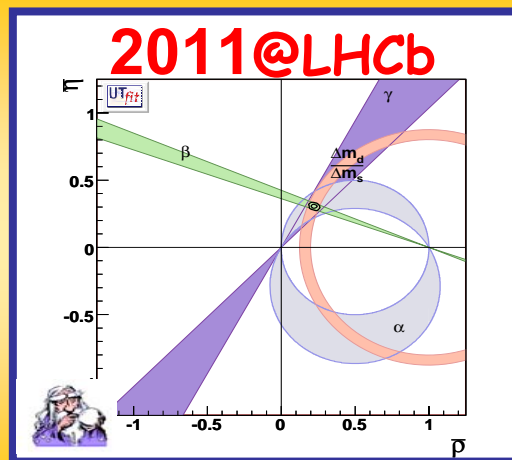
Ciuchini et al., 1995:  
 $\text{Sin}2\beta_{\text{UTA}} = 0.65 \pm 0.12$

Ciuchini et al., 2000:  
 $\Delta m_s = (16.3 \pm 3.4) \text{ ps}^{-1}$

the  
present



and the  
future



# 1. FLAVOUR PHYSICS AND ITS MOTIVATIONS

- Introduction to Flavour Physics
- Evidence of physics beyond the Standard Model
- Flavour physics as a probe of New Physics

# 2. FLAVOUR PHYSICS AND LATTICE QCD

- The “precision era” of Lattice QCD (why now)
- $V_{us}$  and the first row unitarity test
- Lattice QCD and the Unitarity Triangle Analysis
- The UTA beyond the Standard Model:  
evidence of New Physics in  $B_s$  mixing
- Lattice QCD and quark masses



# THE QUARK MASS MATRICES

$$\mathcal{L}_{\text{Yukawa}} = - \sum_{i,k} [\bar{Q}_L^i Y_{ik}^d D_R^k H + \bar{Q}_L^i Y_{ik}^u U_R^k H^c] + \text{h.c.}$$

Gauge symmetry breaking

$$\mathcal{L}_{\text{mass}} = - \sum_{i,k} [\bar{d}_L^i m_{ik}^d d_R^k + \bar{u}_L^i m_{ik}^u u_R^k] + \text{h.c.}$$

$$m^q = Y^q v / \sqrt{2}$$



$$M_W = gv/2$$

Why  $m^q \neq O(M_W)$ ??

# DIAGONALIZATION OF THE MASS MATRIX

The mass matrices  $\mathbf{m}^q$  are not Hermitean. Up to singular cases, they can be diagonalized by 2 unitary transformations:

$$\mathbf{U}_L^\dagger \mathbf{m} \mathbf{U}_R = \mathbf{m}_D$$

$$\left\{ \begin{array}{l} \mathbf{U}_L^\dagger \mathbf{m} \mathbf{m}^\dagger \mathbf{U}_L = \mathbf{m}_D \mathbf{m}_D^\dagger \\ \mathbf{U}_R^\dagger \mathbf{m}^\dagger \mathbf{m} \mathbf{U}_R = \mathbf{m}_D^\dagger \mathbf{m}_D \end{array} \right. \quad i$$

$$(\mathbf{U}_L^\dagger)_{ik} q_L^k \rightarrow q_L^i, \quad (\mathbf{U}_R^\dagger)_{ik} q_R^k \rightarrow q_R^i$$

$$\left( \begin{array}{l} \mathbf{U}_{L,R} \text{ different} \\ \text{for } u^k \text{ and } d^k \end{array} \right)$$

$$\mathcal{L}_{\text{mass}} = - [m_u \bar{u}_L u_R + m_d \bar{d}_L d_R + \dots] + \text{h.c.}$$

With respect to:

$$(\mathbf{U}_L^\dagger)_{ik} q_L^k \rightarrow q_L^i, \quad (\mathbf{U}_R^\dagger)_{ik} q_R^k \rightarrow q_R^i$$

**neutral currents**  $\bar{q}_L^i \gamma_\mu q_L^i$  and  $\bar{q}_R^i \gamma_\mu q_R^i$  are invariant:  
quark kinetic terms, QCD couplings with gluons, QED couplings with photons, weak couplings with  $Z^0$

**No flavor changing neutral currents (FCNC) at tree level**

The only effect is in the **weak charged currents**:

$$\bar{u}_L^i \gamma_\mu d_L^i \cdot W^\mu \rightarrow \bar{u}_L^k \gamma_\mu (\mathbf{U}_L^{u\dagger} \mathbf{U}_L^d)_{kj} d_L^j \cdot W^\mu$$

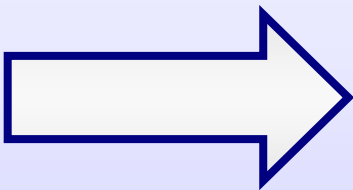
$$\mathbf{V}_{\text{CKM}} = \mathbf{U}_L^{u\dagger} \mathbf{U}_L^d$$

$$\mathbf{V}_{\text{CKM}} \mathbf{V}_{\text{CKM}}^\dagger = \mathbf{1}$$



# THERE IS A CLEAR CORRELATION BETWEEN MASSES AND MIXINGS ANGLES

In the first 2 generations:  $\left(\frac{m_d}{m_s}\right)^{1/2} \approx 0.24$      $\left(\frac{m_u}{m_c}\right)^{1/4} \approx 0.22$



$$\left(\frac{m_d}{m_s}\right)^{1/2} \approx \left(\frac{m_u}{m_c}\right)^{1/4} \approx V_{us}$$

Can we explain this relation ?

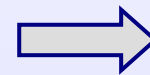
# MASS TEXTURES

Two generations:

Gatto et al.

$$\mathbf{m}^d = m_s \begin{pmatrix} 0 & -\sqrt{x} \\ \sqrt{x} & 1+x \end{pmatrix} \quad \mathbf{m}^u = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix}$$

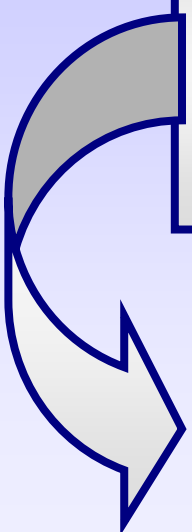
$$\text{diag}(\mathbf{m}^d) = m_s (x, 1)$$



$$x = m_d / m_s$$

Diagonalization:

$$\begin{cases} \mathbf{U}_L^\dagger \mathbf{m} \mathbf{m}^\dagger \mathbf{U}_L = \mathbf{m}_D \mathbf{m}_D^\dagger & \mathbf{U}_L^\dagger \mathbf{m} \mathbf{U}_R = \mathbf{m}_D \\ \mathbf{U}_R^\dagger \mathbf{m}^\dagger \mathbf{m} \mathbf{U}_R = \mathbf{m}_D^\dagger \mathbf{m}_D & \mathbf{V}_{\text{CKM}} = \mathbf{U}_L^{u\dagger} \mathbf{U}_L^d \end{cases}$$


$$V_{\text{CKM}} = U_L^{\text{u}\dagger} U_L^{\text{d}} = U_L^{\text{d}} \approx \begin{pmatrix} 1 - x/2 & \sqrt{x} \\ -\sqrt{x} & 1 - x/2 \end{pmatrix}$$

$$V_{\text{us}} = \sin \theta_C = \sqrt{x} = \sqrt{m_{\text{d}}/m_{\text{s}}} \approx 0.22$$

Which **theory of flavor**  
generates this texture?

# HORIZONTAL SYMMETRIES

Example: **Horizontal U(2)** (Barbieri, Hall, ...)

$$q^a \rightarrow U_{ab} q^b, \quad U \in U(2) \quad a,b = 1,2 \quad \left[ \begin{array}{l} \text{Generation} \\ \text{indices} \end{array} \right]$$

$$L = \frac{1}{M_F} \phi_{ab} q^a q^b H$$

Non-renorm. interaction  
 $M_F =$  flavor scale

"Flavon" field

Higgs field (U(2) scalar)

$$\phi_{ab} = S_{ab} + A_{ab}$$

Symmetric tensor      Anti-symmetric tensor

$$U(2) \xrightarrow{S_{ab}} U(1) \xrightarrow{A_{ab}} \{1\}$$

$$\langle S_{ab} \rangle = \begin{pmatrix} 0 & 0 \\ 0 & V \end{pmatrix} \quad \langle A_{ab} \rangle = \begin{pmatrix} 0 & -v \\ v & 0 \end{pmatrix}$$

$$L = \frac{1}{M_F} (S_{ab} + A_{ab}) q^a q^b H \longrightarrow$$

Flavor symm.  
breaking

$$\longrightarrow \frac{V}{M_F} q^2 q^2 H + \frac{v}{M_F} (q^2 q^1 - q^1 q^2) H \equiv q^a Y_{ab} q^b H$$

Yukawa matrix

$$Y_{ab} = \begin{pmatrix} 0 & -v/M_F \\ v/M_F & V/M_F \end{pmatrix}$$

$$v/M_F = \sqrt{x}$$

$$V/M_F = 1+x$$

Is the Gatto's  
texture