PROBLEM ON PARTICLES & SYMMETRIES

A four-body reaction

(The treatments of the two parts are independent.)

Part I

1. For a given model, the covariant Lagrangian density contains the following terms [1] involving the scalar field ϕ_a,

$$\mathcal{L}_{\phi_a} = \frac{1}{2}\partial_\mu\phi_a\partial^\mu\bar{\phi}_a - \frac{m^2}{2}\phi_a\bar{\phi}_a - g\,\bar{\phi}_b A_\nu\partial^\nu\phi_a - \frac{1}{\Lambda}\phi_a\partial_\nu A^\nu\,\partial^\mu B_\mu\ + \{\text{C.c.}\}\ ,$$

   μ, ν being *Lorentz* indices running from 0 to 3, ∂_μ the **4-vector** derivation, ϕ_b another wave function ($\bar{\phi}_b$ its complex conjugate) for a spin-0 particle and A_μ, B_μ spin-1 fields which behave as **4-vectors** in the covariant formalism. The acronym 'C.c.' means 'Complex conjugate' and concerns each term.

   (a) What is the dimension, and hence the nature, of the parameters m, g and Λ? Demonstrate the answer.

   (b) Show that $\mathcal{L}_{\phi_a}$ is a *Lorentz* invariant.

2. Interpret diagrammatically each term of $\mathcal{L}_{\phi_a}$.

3. Apply the following *Euler-Lagrange* equation, $\frac{\partial\mathcal{L}_{\phi_a}}{\partial\phi_a} = \partial_\mu\frac{\partial\mathcal{L}_{\phi_a}}{\partial[\partial_\mu\phi_a]}$. Comment the resulting equation.

Part II

We consider the four-body reaction, $S_a^- S_a^+ \to S_a^- S_a^+ S_b^- S_b^+$, where the initial and final state particles constitute two species $[a, b]$ of massive scalar (spinless) fields with charges ± 1 (clearly indicated as exponents) under a certain gauge $\mathrm{U}(1)$ symmetry. Within the relativistic quantum framework, this process is induced by the exchange of two neutral V vector (spin-one) bosons, as depicted in the following *Feynman* diagram.

[1] We work within the natural unit system where: $\hbar = c = 1$.

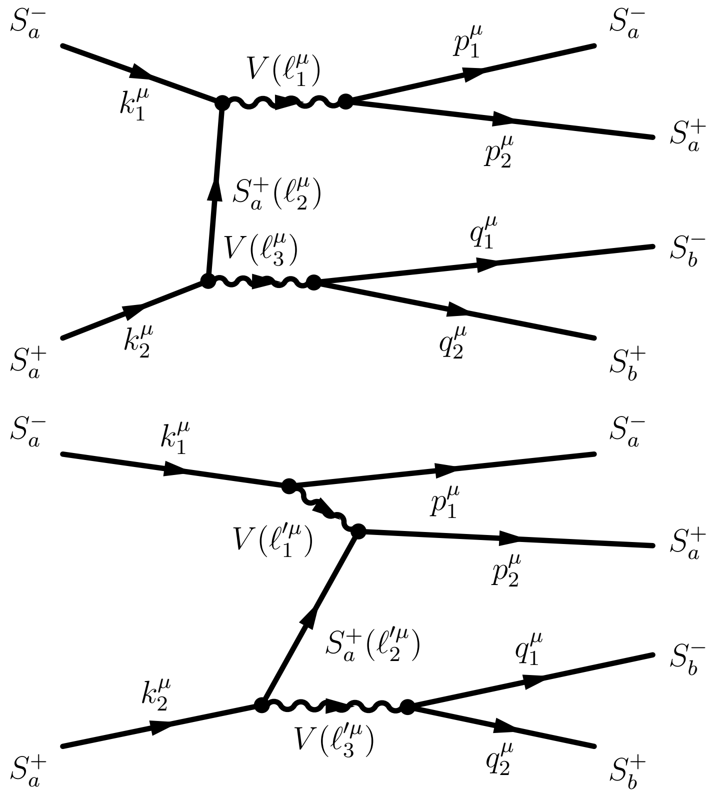

Figure 1: *Feynman* diagrams for the studied 2→4-body reaction. The 4-momentum (k_1^μ, k_2^μ, p_1^ν, p_2^ν, $\ell_1^{(\prime)\rho}$, $\ell_2^{(\prime)\rho}$, $\ell_3^{(\prime)\rho}$, q_1^σ or q_2^σ) associated to each particle is indicated [the greek indices are *Lorentz* indices such that for instance $\mu = 0, 1, 2, 3$]. The arrows along the legs show the propagation flow directions.

1. Make short comments about the charge flows at the interaction vertices and globally for the considered elementary particle reaction.

2. Based on the anti-particle prescription, redraw the *Feynman* diagrams of Figure 1, but considering instead only positively charged scalar particles. Indicate the particle names, all scalar particle direction arrows but only the 4-momenta modified with respect to Figure 1 and specify the two axes orienting the plane of your *Feynman* diagram.

3. Thanks to the previous question, write [2] the probability amplitude, $-i\mathcal{M}$, for the reaction $S_a^- S_a^+ \to S_a^- S_a^+ S_b^- S_b^+$ in terms of the 4-momenta involved in the diagrams of Figure 1, the respective masses $m_{a,b}$ of $S_{a,b}^+$ and the real coupling constant g of the theory. For simplicity we neglect the M_V boson mass terms in the propagators.

4. Make a complete comment about the possibility for the scalar propagator denominators, entering $-i\mathcal{M}$, to vanish.

5. Using the diagrams, express the 4-momenta $\ell_{1,2,3}^\mu$ and $\ell_{1,2,3}'^\mu$ in terms of $p_{1,2}^\mu$, $k_{1,2}^\mu$. Express also $k_1^\mu + k_2^\mu$ as a function of $p_{1,2}^\mu$, $q_{1,2}^\mu$. [3] What are the physical principles invoked? Compare ℓ_2^μ and $\ell_2'^\mu$. Compare ℓ_3^μ and $\ell_3'^\mu$.

[2] Apply directly the provided table of *Feynman* rules in the case where the V interaction to $S_i^\pm S_j^\pm$ does not change the species: $i = j$. Recall that the scalar propagator is $i/(p^\mu p_\mu - m^2)$ for a 4-momentum p^μ and a mass m, and, that the whole amplitude must be *Lorentz* invariant.

[3] Please circle (or frame) clearly the results.

6. Based on previous question, express the following *Lorentz* products (simplifying the obtained results thanks to the nature of external particles),
   (a) $(\ell_2^\nu - k_1^\nu)(p_{2\nu} - p_{1\nu})$ in terms of $p^\mu_{1,2}$, k^μ_1,
   (b) $(\ell_2^\nu + k_2^\nu)(q_{2\nu} - q_{1\nu})$ in terms of $q^\mu_{1,2}$, k^μ_2,
   (c) $(p_1^\nu + k_1^\nu)(p_{2\nu} + \ell'_{2\nu})$ in terms of $p^\mu_{1,2}$, k^μ_1.
7. Use the Questions n'5 and 6 to conclude on the equality (or not) between the two terms of $-i\mathcal{M}$. Comment.
8. Calculate [4] the *Lorentz* product $p_1^\nu p_{2\nu}$ between the 4-momenta $p_1^\nu = (E_1, \vec{p}_1)$ and $p_2^\nu = (E_2, \vec{p}_2)$. Provide the result in terms of $E_1, \vec{p}_1, E_2, \vec{p}_2$.
9. Draw the two other *Feynman* diagrams contributing to the considered reaction but not shown on Figure 1. Indicate the particle names ($S^\pm_{a,b}, V$) and propagation arrows but not the 4-momenta.

***

[4] With the usual metric $g^{\alpha\beta} = diagonal(+1, -1, -1, -1)$.