PROBLEM ON PARTICLES & SYMMETRIES

The three-body decay

Let us consider an interaction which induces the decay of a spinless particle (scalar field) with charge -1, denoted here S_a^-, into three other species of spinless particles with charge ± 1, namely S_b^-, S_c^- and S_d^+. This reaction is generated [1] by the exchange of a neutral spin-one boson V. Such a decay process is depicted on the following Feynman diagram.

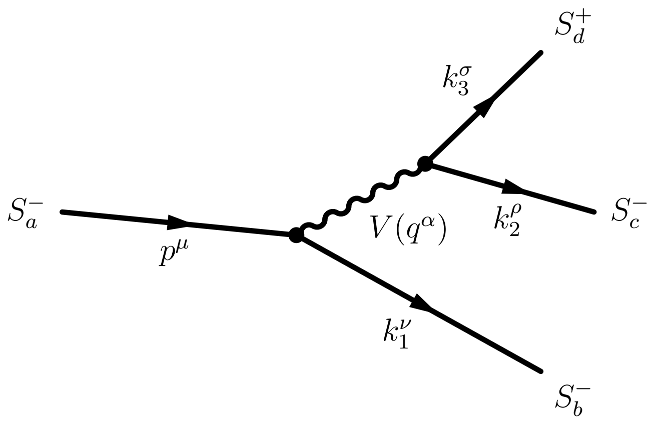

Figure 1: Feynman diagram for the three-body decay. The 4-momentum q^α, p^μ, $k^\sigma_{1,2,3}$ associated to each particle is indicated [α, μ, σ are Lorentz indices such that for example $\mu = 0, 1, 2, 3$].

1. What are the two axes of the plane on which the Feynman diagram of Figure 1 is represented?

2. Make a comment about the charge conservation in the studied decay process.

3. Using the anti-particle prescription, draw the Feynman diagram for the same decay process, but using instead the S_c^+ propagation. Indicate the corresponding 4-momentum along the S_c^+ leg.

4. By applying directly the Feynman rules (presented during the lectures), write the probability amplitude $\mathcal{M}$ for the reaction of Figure 1 in terms of the 4-momenta q^α, p^μ, $k^\sigma_{1,2,3}$ and the coupling constant g. Consider the most general case of a V boson with mass M_V and comment [2] about the possibility for the propagator denominator to vanish.

[1] The considered interaction is the same as the one described by the spinless Quantum Electro-Dynamics theory but it couples different particle species.

[2] Mathematically and physically.

5. Express the 4-momentum q^α as a function of k_2^ν, k_3^σ, and, give also p^α in terms of k_1^ρ, k_2^ν, k_3^σ. Which physical principle has allowed you to obtain those results?
6. Express the amplitude $\mathcal{M}$, found in question 4, in terms of g, M_V and $k^\sigma_{1,2,3}$ exclusively [3].
7. From now on, we will assume that the three final particles are in the high-energy regime. Then show, for instance for the particle with 4-momentum $k_1^\mu \equiv (E_1, \vec{k}_1)$, that, $E_1 \simeq k_1$, where E_1 denotes its global energy and $k_1 = ||\vec{k}_1||$ its momentum norm [4].
8. Calculate $k_1^\mu k_{2\mu}$ and then $k_1^\mu k_{1\mu}$, in the high-energy limit where masses are neglected. Express $k_1^\mu k_{2\mu}$ only in terms of k_1, k_2 and θ_{12}, the angle between $\vec{k}_1$ and $\vec{k}_2$.
9. Based on the previous question, express the following quantities in terms of $k_{1,2,3}$, $c_{12} = \cos\theta_{12}$ as well as c_{23}, c_{13}.
   (a) $(k_3^\mu + k_2^\mu)(k_{3\mu} + k_{2\mu})$.
   (b) $(k_3^\mu - k_2^\mu)(k_{3\mu} + k_{2\mu})$.
   (c) $(k_3^\mu - k_2^\mu)(k_{3\mu} + k_{2\mu} + 2k_{1\mu})$.
10. Simplify the obtained amplitude thanks to the analytical results of questions 9a-9b-9c (the final particle masses are also neglected with respect to M_V). Provide $\mathcal{M}$ as a function of g, M_V, $k_{1,2,3}$ and c_{12}, c_{23}, c_{13}, only.

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[3] To get more compact notations, one may use the definition, $p.q \mathrel{\hat{=}} p^\mu q_\mu$, for covariant products and, $p^2 \mathrel{\hat{=}} p^\mu p_\mu$, for the Lorentz square.

[4] Throughout all the exercise, the light velocity c is taken equal to unity for simplicity reasons (natural units) and we consider the usual metric $g^{\alpha\beta} = diagonal(+1, -1, -1, -1)$.