PROBLEM ON PARTICLES & SYMMETRIES

Vector boson fusion

We consider the three-body reaction $S_a^- S_b^+ \to S_a^- S_b^+ S_c^+$, where the initial and final state particles constitute three species $[a, b, c]$ of massive scalar (spinless) fields with charges ± 1 (clearly indicated as exponents) under a certain gauge $\mathrm{U}(1)$ symmetry. Within the relativistic quantum framework, this process is induced by the exchange of two neutral V vector (spin-one) bosons, as depicted in the following *Feynman* diagram.

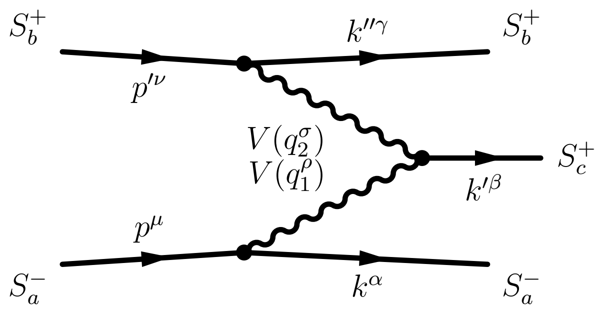

Figure 1: *Feynman* diagram for the studied two to three-body reaction. The 4-momentum (p^μ, p'^ν, q_1^ρ, q_2^σ, k^α, k'^β, k''^γ) associated to each particle is indicated [the greek indices are *Lorentz* indices such that for instance $\mu = 0, 1, 2, 3$]. The arrows show the propagation flow directions.

1. Make a comment about the charge flow associated to the particles S_a^- and S_b^+ on Figure 1.
2. Based on the anti-particle prescription, draw the *Feynman* diagram for the process of Figure 1, but considering instead the S_a^+ scalar propagation. Indicate the corresponding 4-momenta along the external scalar legs and specify the two axes orienting the plane of your *Feynman* diagram.
3. Write [1] the probability amplitude, $-i\mathcal{M}$, for the reaction $S_a^- S_b^+ \to S_a^- S_b^+ S_c^+$ in terms of the 4-momenta p^μ, p'^ν, q_1^ρ, q_2^σ, k^α, k''^γ and the real coupling constant g of the theory. The *Feynman* rules dictate that a *Lorentz* product must be taken between one *Lorentz* index from the $V(q_1^\rho)$ propagator and one *Lorentz* index from the $V(q_2^\sigma)$ propagator. For simplicity we neglect the M_V boson mass terms in the propagator numerator (only).
4. Make a complete comment [2] about the possibility for the propagator denominator, entering $-i\mathcal{M}$, to vanish.

[1] Apply directly the provided table of *Feynman* rules in the case where the V interaction to $S_i^\pm S_j^\pm$ does not change the species: $i = j$. Recall that the whole amplitude must be *Lorentz* invariant.

[2] Mathematically and physically.

5. Express the 4-momentum q_2^μ, first in terms of p'^α and k''^β and secondly in terms of q_1^α and k'^β. Then express p^ν as a function of k^α, k'^β, k''^γ and p'^δ. Justify.

6. Based on previous question, give $-i\mathcal{M}$ exclusively in terms of p'^ν, q_1^ρ, q_2^σ, k^α, k'^β, k''^γ and g.

7. Show that in the high-energy regime for initial and final particles, the relativistic energy, entering a 4-momentum $k^\alpha = (\frac{E_k}{c}, \vec{k})$, reads at zeroth order as $E_k \simeq kc$, where $k = ||\vec{k}||$. At this order, deduce [3] the 4-momentum *Lorentz* products $k_\mu k^\mu$ and $k'_\mu k^\mu$ [in terms of k, k' and the angle denoted $(\vec{k}, \vec{k}')$ between $\vec{k}$ and $\vec{k}'$].

8. At the same relativistic order as in previous question, calculate the 4-momentum product $(2k^\mu + k'^\mu + k''^\mu - p'^\mu)(p'_\mu + k'_\mu)$ in terms of p', k, k', k'' and angles among 3-momenta.

9. Still at the same relativistic order, calculate the 4-momentum products $q_1^\mu q_{1\mu}$ and $q_2^\mu q_{2\mu}$ as functions of p, p', k, k'' and angles among 3-momenta.

10. The relevant part of the effective Lagrangian density involving the complex scalar field $\phi(x^\mu)$ for the particle S_c^+ reads as,

$$\mathcal{L}_\phi = \frac{1}{2} D_\mu \phi (D^\mu \phi)^* + g M_V A_\mu A^\mu \phi + g M_V A_\mu A^\mu \phi^* \ ,$$

where the exponent star indicates the complex conjugate, $D_\mu = \partial_\mu + igA_\mu$, $\partial_\mu = \frac{\partial}{\partial x^\mu}$ and $A^\mu(x^\nu)$ represents the real vector field for the V boson.

   (a) Interpret physically, in a few words/drawings, each term of $\mathcal{L}_\phi$ (with the help of Figure 1).

   (b) Derive [4] the equation of motion for the ϕ^* field from $\mathcal{L}_\phi$. Comment.

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[3] Let us consider the usual metric $g^{\alpha\beta} = diagonal(+1, -1, -1, -1)$.

[4] Using the *Euler-Lagrange* equation form $\frac{\partial \mathcal{L}}{\partial \phi} = \partial_\mu (\frac{\partial \mathcal{L}}{\partial [\partial_\mu \phi]})$.