PROBLEM OF PARTICLE PHYSICS

Dirac matrices

1. **Lagrangian hermiticity.-** In the relativistic quantum theory, the Lagrangian density for a free massive spin-1/2 field, with mass m, reads as,

$$\mathcal{L} \;=\; i\,\bar{\psi}\gamma^\mu\partial_\mu\psi - m\,\bar{\psi}\psi\;.$$

   This Lagrangian is written under a covariant form where μ stands for the *Lorentz* index: $\mu = 0, 1, 2, 3$.

   (a) Calculate the Hermitian conjugate of the second term, $-m\bar{\psi}\psi$, for a real mass.

   (b) Calculate the Hermitian conjugate of the first term, $i\bar{\psi}\gamma^\mu\partial_\mu\psi$. Using standard *Dirac* matrix relations, express the result without explicit γ^0 matrices, and, in terms of γ^μ only (instead of $\gamma^{\mu\dagger}$).

   (c) Recast the resulting term of the previous question into a total derivative term plus another term [1]. What is the contribution of the total derivative term to the action, $\mathcal{A} = \int d^4x\;\mathcal{L}$?

   (d) Conclude about the hermiticity of the whole considered Lagrangian.

   (e) Calculate the Hermitian conjugate of a possible axial-vector bilinear quantity: $(\bar{\psi}\gamma^\mu\gamma^5\psi)^\dagger$, in term of, $\bar{\psi}\gamma^\mu\gamma^5\psi$.

2. **Charge conjugation.**

   (a) Calculate the bilinear quantity involving the charge conjugate spinors, $\overline{\psi_C}\psi_C$, in terms of ψ^t and $\bar{\psi}^t$.

   (b) The result of previous question being a number (scalar product of spinors), it is equal to its self-transposed. Use this property to express it in term of $\bar{\psi}\psi$.

   (c) Calculate now the bilinear quantity, $\overline{\psi_C}\gamma^\mu\psi_C$, still in terms of ψ^t and $\bar{\psi}^t$ (getting rid of explicit C operators in the final result).

   (d) Once more, transpose the result of previous question to express it in term of $\bar{\psi}\gamma^\mu\psi$.

3. $\not{p}$ **product.-** Using the relation about anti-commutation of *Dirac* matrices, demonstrate that $\not{p}\not{p} = p_\mu p^\mu\;\mathbb{1}_{4\times4}$, where $\not{p} \hat{=} \gamma_\mu p^\mu$ involves the 4-momentum p^μ.

***

[1] One could possibly describe this operation as an integration by part in the action.