M1 General Physics **2020-2021**
Major PNU

Particles

Final exam

January 4th 2021

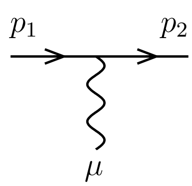
Documents allowed

Amplitudes

In the whole problem, “electron” and “positron“ should be understood as scalar particle of charge $e = -|e|$ and $|e|$ respectively.

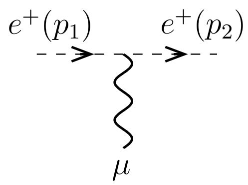
0. Preliminary

(a) The feynman rule for the spinless electron-photon vertex is known to be



$= ie(p_1 + p_2)^\mu\,.$

Explain why the feynman rule for the spinless positron-photon vertex is



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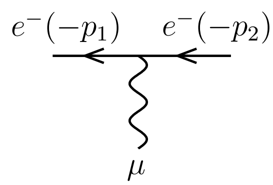

$= ie(-p_1 - p_2)^\mu\,.$

1. We consider the process

$$e^-(p_A)\, e^-(p_B) \to e^-(p_C)\, e^-(p_D)\,. \tag{1}$$

(a) At lowest order in perturbation theory, how many Feynman diagrams can be drawn? Draw them.

(b) Write the scattering amplitude $\mathcal{M}^{e^-e^-\to e^-e^-}$ of this process.

2. We consider the process

$$e^-(p_A)\, e^+(p_B) \to e^-(p_C)\, e^+(p_D)\,. \tag{2}$$

(b) At lowest order in perturbation theory, how many Feynman diagrams can be drawn? Draw them, using only electron lines, relying on the antiparticle prescription.

(c) Write the scattering amplitude $\mathcal{M}_{e^-e^+\to e^-e^+}$ of this process.

3. Due to the antiparticle prescription, we know that for an arbitrary particle P, the set $P(p)\bar{P}(-p)$ is the same as the vacuum. Starting from an arbitrary $2 \to 2$ process generically written as

$$A(p_A)\, B(p_B) \to C(p_C)\, D(p_D) \tag{3}$$

and adding on the left hand side a particle-antiparticle pair of a suitable type, and on the right hand side another particle-antiparticle pair of different type, show that this process is equivalent to the process

$$A(p_A)\, \bar{D}(-p_D) \to C(p_C)\, \bar{B}(-p_B) \tag{4}$$

and thus that

$$\mathcal{M}^{AB\to CD}(p_A, p_B, p_C, p_D) = \mathcal{M}^{A\bar{D}\to C\bar{B}}(p_A, -p_D, p_C, -p_B)\,, \tag{5}$$

a property known under the name of crossing symmetry.

4. Compare the two amplitudes

$$\mathcal{M}^{e^-e^+\to e^-e^+}(p_A, p_B, p_C, p_D)$$

and

$$\mathcal{M}^{e^-e^-\to e^-e^-}(p_A, -p_D, p_C, -p_B)\,.$$

Comment and explain why this should be expected.

5. We introduce the three Mandelstam variables

$$\begin{aligned} s &= (p_A + p_B)^2 && (6)\\ t &= (p_A - p_C)^2 && (7)\\ u &= (p_A - p_D)^2. && (8) \end{aligned}$$

(a) Show that we also have

$$\begin{aligned} s &= (p_C + p_D)^2 && (9)\\ t &= (p_B - p_D)^2 && (10)\\ u &= (p_C - p_B)^2. && (11) \end{aligned}$$

(b) Translate the crossing discussed in question 4. in terms of the exchange of two variables among s, t, u. Deduce a relation between the amplitudes of the two processes when expressed as functions of s, t, u.

(c) Find another crossed reaction involving the exchange of another subset of two variables among s, t, u, and provide the relation between the two amplitudes, expressed as functions of momenta, and then expressed as functions of Mandelstam variables.

6. Explicit expressions of the amplitudes and crossing properties

(a) Compute the scattering amplitude of the process (1) as a function of e^2, s, t, u.

(b) Compute the scattering amplitude of the process (2) as a function of e^2, s, t, u.

(c) Crossing properties:

(i) Comment on s, t, u crossing properties of the two diagrams involved in the process (1).

(ii) Comment on s, t, u crossing properties of the two diagrams involved in the process (2).

(iii) Comment on s, t, u crossing properties between the two processes (1) and (2).

7. Kinematics in the center-of-mass frame.

We now consider the center-of-mass frame, and we denote $\vec{p}_i = \vec{p}_A = -\vec{p}_B$ and $\vec{p}_f = \vec{p}_C = -\vec{p}_D$, and $p_i^* = |\vec{p}_i|$ and $p_f^* = |\vec{p}_f|$.

(a) Explain why $p_A^0 = p_B^0 = \sqrt{s}/2$ and $p_C^0 = p_D^0 = \sqrt{s}/2$.

(b) Show that $p_i^* = p_f^*$.

(c) We denote $k = p_i^* = p_f^*$ and introduce the scattering angle θ, i.e. the angle between $\vec{p}_i$ and $\vec{p}_f$.

(i) Show that

$$s = 4m^2 + 4k^2 \,. \tag{12}$$

(ii) Show that

$$t = -2k^2(1 - \cos\theta) \,. \tag{13}$$

(iii) Show that

$$u = -2k^2(1 + \cos\theta) \,. \tag{14}$$

8. Cross-sections

(a) Write the differential cross-section $d\sigma/d\Omega$ in the center-of-mass frame for the process (1) as a function of s, t, u, introducing the fine structure constant

$$\alpha_{em} = \frac{e^2}{4\pi}.$$

(b) Write the differential cross-section $d\sigma/d\Omega$ in the center-of-mass frame for the process (2) as a function of s, t, u.

(c) Write the two above cross-sections as functions of k^2 and $\cos\theta$.

(d) Comment on the behavior of the cross-section for the process $e^-e^- \to e^-e^-$ when $\theta \to 0$ or $\theta \to \pi$. What is the technical origin of this? Can one find a physical explanation?