M1 General Physics **2021-2022**
Major PNU

Particles

Final exam: session 2

March 24th 2022

Documents allowed

Notes:
- Space coordinates may be freely denoted as (x, y, z) or (x^1, x^2, x^3).
- One may always assume that fields are rapidly decreasing at infinity.

1 Muon decay

A muon is a heavy lepton, of mass $m_\mu = 105$ MeV. Its mean life-time is $\tau = \frac{1}{\lambda} = 2.2\,10^{-6}$s. Muons are created in the upper atmosphere when cosmic rays collide with air molecules.

1. Consider a muon of energy 17 GeV. What fraction of the light velocity does it carry, as seen by an observer on Earth?

2. What is the mean life-time of such a muon, again as seen by an observer on Earth?

3. Out of a million particles produced at altitude 50 km with the above energy, how many will reach the Earth before decaying?

4. Compare this result with the one obtained in a non-relativistic treatment. Comment.

2 Nonrelativistic Lagrangian

The complex scalar field $\psi(\vec{r}, t)$ in the nonrelativistic approximation has a Lagrangian

$$\mathcal{L} = \frac{i\hbar}{2}\left(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}\right) - \frac{\hbar^2}{2m}|\vec{\nabla}\psi|^2 - U|\psi|^2 \tag{1}$$

where $m > 0$ is the particle mass, $\hbar$ the reduced Planck constant and $U(\vec{r}, t)$ is the potential field in which the particle propagates.

1. Derive the two equations of motion for $\psi(\vec{r}, t)$ and $\psi^*(\vec{r}, t)$ from the Lagrangian (1) and interpret them.

2. Hamiltonian
a. Construct the Hamiltonian function density $\mathcal{H}$ using the Lagrangian.

b. Calculate the total field energy and comment.

c. Propose the quantum interpretation of the result thus obtained, in the case of a time independent potential.

3. The Schrödinger equation for the wave function $\psi(\vec{r}, t)$ of a spin-free nonrelativistic particle of charge q has the form

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \left[\vec{\nabla} - \frac{iq}{\hbar c} \vec{A}(\vec{r}, t) \right]^2 \psi + q\,\varphi(\vec{r}, t)\,\psi \tag{2}$$

where $\vec{A}(\vec{r}, t)$ and $\varphi(\vec{r}, t)$ are the electromagnetic potentials (specified real functions).

a. Guess the Lagrangian leading to (2).
Hint: rely on the covariant derivative $\vec{D} = \vec{\nabla} - \frac{iq}{\hbar c} \vec{A}$ and on the guess of the potential U.

b. Show in detail that the equation of motion of this Lagrangian is indeed the Schrödinger equation (2).