M1 General Physics **2022-2023**

Particles

Exam

Second session

February 9th 2023

Documents allowed

Notes:

- **The subject is deliberately long**. Solving at least one of the two problems will ensure a good mark!
- One may use the usual system of units in which $c = 1$ and $\hbar = 1$.
- Space coordinates may be freely denoted as (x, y, z) or (x^1, x^2, x^3).
- Any drawing, at any stage, is welcome, and will be rewarded!

1 Photo-production of charm

The lightest meson containing a charmed quark is the D^0. The production of a D^0 meson and of its anti-particle $\bar{D}^0$ can be done by using a beam of high energy photons which collide with protons (immobile in the reference frame of the laboratory R) according to the reaction

$$\gamma\, p \to p\, D^0\, \bar{D}^0\,. \tag{1}$$

We denote as m_p the proton mass and m_0 the D^0 mass (which is identical to the mass of the $\bar{D}^0$).

1. We seek to determine the reaction threshold, i.e. the minimum energy of the photon for which the reaction can take place. We will note E_γ the value of this energy in the laboratory reference frame.

(i) Recall the definition of the center of mass reference frame R^*.

(ii) At threshold, the momentum in R^* of each produced particle vanishes. Write in R^* the sum of the incoming energies $E^*_\gamma + E^*_p$ as a function of m_p and m_0.

(iii) Compute $(p_\gamma + p_p)^2$ in both R and R^* frames and deduce the value of E_γ as a function of m_p and m_0.

(iv) Compute numerically E_γ.

We give $m_p = 938$ MeV/c^2 and $m_0 = 1865$ MeV/c^2.

2. We want to create a beam of very energetic photons. For this we use the Compton back-scattering : a beam of electrons of 30 GeV collides head-on with a monochromatic beam of monochromatic beam of photons of wavelength $\lambda_1 = 266$ nm (a laser). The kinematics of the

process is represented on the figure below in the laboratory frame R as well as in the frame R' in which the electron (bold point) is initially at rest. The incident photon is designated by 1 and the scattered photon by 2.

(i) Write the conservation of the quadri-momentum in R'. Deduce the expression of the energy E_e' of the scattered electron as a function of the energy E_1' of the incoming photon, of the energy E_2' of the scattered photon and of the mass m_e of the electron.

(ii) Show that one has the following relation in R':

$$E_2' = \frac{E_1'}{1 + \frac{E_1'}{m_e}(1 - \cos\theta')} \,. \tag{2}$$

(iii) Express the Lorentz factor γ when passing from the frame R to R', and compute its numerical value.

Compute the numerical values of E_1, E_1' and $E_2'(\theta' = \pi)$.

We give $m_e = 0.511$ MeV/c^2 and $h = 6.626 \cdot 10^{-34}$ J.s.

3. Backscattering

(i) Justify that $\cos\theta = -p_{x2}/E_2$, and write a similar relation in R'. Using the Lorentz transformation allowing to pass from R to R', deduce that

$$\cos\theta = \frac{\cos\theta' - \beta}{1 - \beta\cos\theta'} \,. \tag{3}$$

(ii) In the SLAC setup, deduce that the photons are mainly emitted in the forward region in the laboratory frame ($\theta \sim \pi$.).

(iii) What is the dominant angle of emission in the frame R'?

(iv) Suppose, just for the present question, that γ is arbitrary (therefore the electron may or may not be relativistic in the laboratory frame). If one detects the scattered photon at an angle $\theta = \pi$ in the laboratory frame, what would be the angle θ' in the rest frame of the electron? Comment.

(v) Express the energy E_2 for $\theta = \theta' = \pi$. Compute its numerical value. Comment.

4. Below is the photon energy spectrum produced at SLAC, in an experiment dedicated to charm photoproduction. Comment.

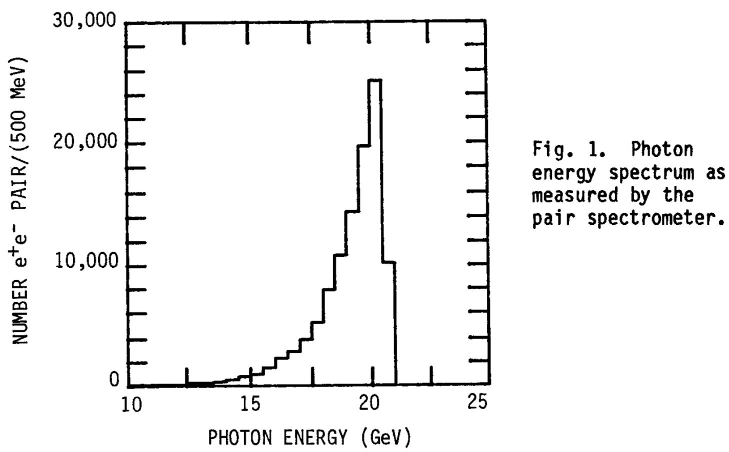
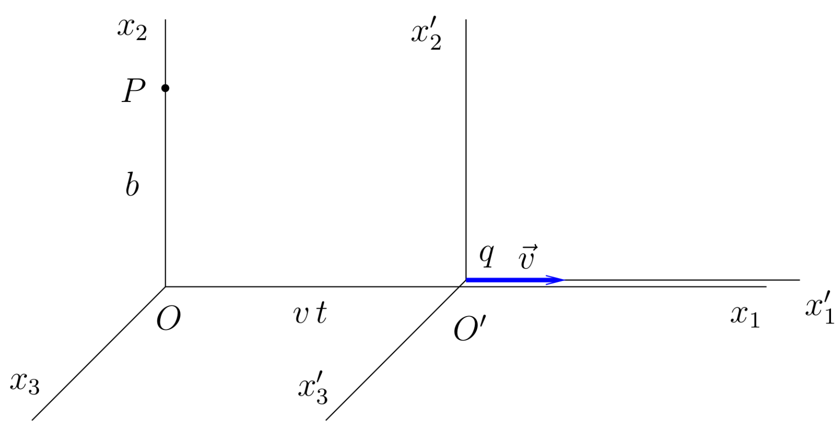


Figure from *AIP Conference Proceedings 113, 419 (1984).*

2 Field of a charge in uniform rectilinear motion

We consider a charge q in uniform rectilinear motion at the speed $\vec{v}$ in the observer's reference frame K. Let us note K' the rest frame of this charge, located at the origin O' of this one. We orientate the frames linked to K and K' so that the axes x_i and x'_i are collinear, with x_1 and x'_1 pointing in the direction of the motion of the charge, and thus $\vec{v} = v$, $(v \geq 0)$. We will note t and t' the times respectively in the reference frames K and K'. We suppose that at $t = t' = 0$, the origins O and O' of the two reference frames coincide. The observer is at a distance b from O in the reference frame K, oriented so that $\overrightarrow{OP} = b,\, \vec{u}_2$.



2.1 Preliminary question:

We consider two inertial reference frames K and K' so that K' is obtained from K by an arbitrary boost of velocity $\vec{v} = \vec{beta} = \vec{n}$. Let us denote $\{\vec{E}, \vec{B}\}$ and $\{\vec{E}', \vec{B}'\}$ the electromagnetic fields respectively in these two reference frames. We recall the following relations allowing us to express $\{\vec{E}', \vec{B}'\}$ using $\{\vec{E}, \vec{B}\}$:

$$\begin{aligned} \vec{E}' &= (\vec{E}\cdot\vec{n})\vec{n} + \gamma\left[\vec{E} - (\vec{E}\cdot\vec{n})\vec{n}\right] + \gamma\,\vec{v}\wedge\vec{B}\,, && (4) \\ \vec{B}' &= (\vec{B}\cdot\vec{n})\vec{n} + \gamma\left[\vec{B} - (\vec{B}\cdot\vec{n})\vec{n}\right] - \gamma\,\vec{v}\wedge\vec{E}\,. && (5) \end{aligned}$$

Express $\{\vec{E}, \vec{B}\}$ as a function of $\{\vec{E}', \vec{B}'\}$.

2.2 Fields

1. Show that in K', the electromagnetic fields at point P can be written as

$$\begin{aligned} E'_1 &= -\frac{qvt'}{4\pi r'^3}\,, && (6) \\ E'_2 &= \frac{qb}{4\pi r'^3}\,, && (7) \\ E'_3 &= 0\,, && (8) \\ \vec{B} &= \vec{0}\,. && (9) \end{aligned}$$

Provide the expression of r' as a function of b and t'.

2. Show that using the coordinates of K, this field also reads

$$\begin{aligned} E'_1 &= -\frac{q}{4\pi}\frac{v\gamma t}{\left(b^2 + v^2\gamma^2 t^2\right)^{3/2}}, && (10) \\ E'_2 &= \frac{q}{4\pi}\frac{b}{\left(b^2 + v^2\gamma^2 t^2\right)^{3/2}}\,. && (11) \end{aligned}$$

3. Show that

$$\begin{aligned} E_1 &= E'_1 = -\frac{q}{4\pi}\frac{v\gamma t}{\left(b^2 + v^2\gamma^2 t^2\right)^{3/2}}, && (12) \\ E_2 &= \gamma E'_2 = \frac{q}{4\pi}\frac{\gamma b}{\left(b^2 + v^2\gamma^2 t^2\right)^{3/2}}\,, && (13) \\ B_3 &= \gamma\beta E'_2 = \beta E_2\,. && (14) \end{aligned}$$

2.3 Non relativistic limit

4. Consider the limit $\gamma \to 1$.

i) Discuss and comment the expression of the electric field $\vec{E}$ in this limit.

ii) Same questions for the magnetic field $\vec{B}$. The result obtained will be interpreted from the point of view of the law of Biot and Savart.

2.4 Study of relativistic effects

5. Time variation of the field transverse to the direction of motion of the particle E_2.

i) Plot the transverse field E_2 as a function of vt, for $\gamma \sim 1$ and $\gamma \gg 1$.

ii) Specify the possible extrema, and their temporal width.

iii) Discuss the change in the shape of E_2 when we go from $\beta \ll 1$ to $\beta \to 1$.

6. Time variation of the longitudinal field E_1.

i) Study and plot the longitudinal field E_1 as a function of vt, for $\gamma \sim 1$ and $\gamma \gg 1$.

ii) Specify the possible extremes.

iii) Discuss the change in the shape of E_1 when we go from $\beta \ll 1$ to $\beta \to 1$.

7. Compare the amplitude of these two fields in the $\beta \to 1$ limit

8. i) At $t = 0$, compare the electric field transverse to the direction of motion of the particle E_2 to its non-relativistic value.

ii) Give an order of magnitude of the duration of the electromagnetic pulse resulting from the passage of the charged particle.

iii) Discuss the effect of the longitudinal field.

iv) For a low temporal resolution (in front of a scale to be specified), show that the $\vec{E}$ field behaves like a plane wave whose structure (polarization and direction of propagation) will be specified.

9. It is assumed that the moving charge is a particle of charge $q = ze$ and that in P is an atomic electron of charge $-e$.

i) From the above, deduce an evaluation of the impulse transferred Δp to the electron during the passage of the mobile charge. Verify that the result is independent of γ.

ii) Calculate this transferred pulse exactly.