M1 General Physics **2022-2023**

Particles

Mid-term exam

October 24th 2022

Documents allowed

Notes:
- One may use the usual system of units in which $c = 1$ and $\hbar = 1$.
- Space coordinates may be freely denoted as (x, y, z) or (x^1, x^2, x^3).
- Any drawing, at any stage, is welcome, and will be rewarded!

1 Particle decay

1.1 Particle decay in the rest frame

We consider an unstable particle of mass M, which can decay in two daughter particles of masses m_1 and m_2.
1. In the center-of-mass frame (CMS), write the 4-momentum P of the decaying particle.

We denote by p the norm of 3-momentum of particles 1 and 2 in the CMS.
2. Express the sum of the two energies E_1 and E_2 of the produced particles in the CMS as a function of p.

3. Show that

$$p = \frac{1}{2M}\sqrt{[M^2 - (m_1 - m_2)^2][M^2 - (m_1 + m_2)^2]}\,. \tag{1}$$

4. Show that

$$M \geq m_1 + m_2\,. \tag{2}$$

5. Show that

$$E_1 = \frac{1}{2M}\left(M^2 + m_1^2 - m_2^2\right) \tag{3}$$

and

$$E_2 = \frac{1}{2M}\left(M^2 + m_2^2 - m_1^2\right)\,. \tag{4}$$

6. Is there any preferred direction for the emitted particles? What can be said for the second particle if the first one is detected in a given direction?

7. Simplify the above results in the case where the two daughter particles are equal, for instance in the decay of a neutral kaon into a pair of pions.

1.2 Particle decay of an unstable particle in flight

We now consider an unstable particle in flight in the laboratory (LAB) frame. For convenience, we chose the $z-$axis as being along the direction of flight of the mother particle. We now denote $\vec{p}$ the 3-momentum of the *mother* particle, and E its energy.
We denote as $\vec{p}_{1\perp}$ and $\vec{p}_{2\perp}$ the transverse momentum (with respect to $z-$axis) of particle 1 and 2 respectively, and p_{1z} and p_{2z} their longitudinal momentum, so that the momenta of daughter particles read

$$
\begin{aligned}
p_1 &= (E_1, \vec{p}_{1\perp}, p_{1z})\,, & (5)\\
p_2 &= (E_2, \vec{p}_{2\perp}, p_{2z})\,. & (6)
\end{aligned}
$$

1. Write P, the 4-momentum of the mother particle, and compare $\vec{p}_{1\perp}$ and $\vec{p}_{2\perp}$, and p, p_{1z} and p_{2z}.

2. We now use asterisks for momenta in the CMS.
Write precisely the boost from the CMS to the LAB frame for the mother particle.

3. Deduce the boost for particle 1 and 2.

4. Show that the LAB angle θ_1 that daughter particle 1 makes with the direction of flight of the mother particle in a two-body decay is related to the CMS angle θ_1^* by the following equation

$$
\tan\theta_1 = \frac{\sin\theta_1^*}{\gamma(\beta/\beta_1^* + \cos\theta_1^*)} \tag{7}
$$

where β is the LAB velocity of the mother particle and β_1^* the CMS velocity of the daughter particle.

2 Train in a tunnel

Preliminary:
Show that in a boost from a frame F in which an object is at rest, to an arbitrary frame F', lengths along the direction of the boost gets contracted by a factor γ, while transverse distances are unaffected.
Hint: consider a rod at rest in a frame F, parallel to the axis of the boost. Determine its length in the boosted frame F', i.e. measured *at one and the same time* t' in this frame F'.

A train and a tunnel both have proper lengths L. The train moves toward the tunnel at speed v. A bomb is located at the front of the train. The bomb is designed to explode when the front of the train passes the far end of the tunnel. A deactivation sensor is located at the back of the train. When the back of the train passes the near end of the tunnel, the sensor tells the bomb to disarm itself. Does the bomb explode? To answer to this question, we will consider two different frames, with two answers which should be obviously identical.

1. Easiest discussion: First consider the frame of the train and conclude.

2. Naively paradoxical result: Consider now the frame of the tunnel. Conclude.
Hint: Escape the paradox by taking into account the time of propagation of any signal.

3 Impossibility of certain processes

1. Show that the process $\gamma \to e^- e^+$ is impossible in the vacuum.

2. Show that this reaction is possible in the vicinity of a heavy nucleus and calculate the threshold energy for this reaction.

3. Show that it is impossible for a free and isolated electron to absorb a photon.