M1 General Physics **2023-2024**

Particles

Mid-term exam

November 7th 2023

Documents allowed

Notes:
- **Exercises are independent**.
- The questions or exercises with a * are more difficult. Any serious trial will be rewarded.
- One may use the usual system of units in which $c = 1$ and $\hbar = 1$.
- Space coordinates may be freely denoted as (x, y, z) or (x^1, x^2, x^3).
- Any drawing, at any stage, is welcome, and will be rewarded!

1 A photon travels at the speed of a photon!

A photon moves at an angle θ with respect to the x' axis in frame S'. Frame S' moves at speed v with respect to frame S (along the x' axis).

1. Calculate the components of the photon's velocity in S, and verify that its speed is c, as it should!

2. Express the frequency ν' of the photon in frame S' in terms of its frequency ν in frame S, the angle θ and the speed v.

3. Discuss the various limits of interest.

2 Photoproduction of a particle

A photon, of frequency ν, collides with a particle of mass M and produces in the final state a particle of mass M', close to M, and a particle of mass m, which is said to be photoproduced by the reaction.

1. What is the minimum frequency required to cause this type of reaction? Application to the photoproduction of charged pions by reaction with protons:

$$\gamma\, p \to n\, \pi^+. \tag{1}$$

2. Simplifies this result in the case $M' = M$, and discuss the case of the photoproduction of π^0, as

$$\gamma\, p \to p\, \pi^0. \tag{2}$$

Compare the minimal frequencies in the two above cases.

Data: $M_p c^2 = 938.272$ MeV, $M_n c^2 = 939.565$ MeV, $m_{\pi^+} c^2 = 139.57$ MeV, $m_{\pi^0} c^2 = 134.976$ MeV, $h = 6.626 \times 10^{-34}$ J·s, 1 eV $= 1.602 \times 10^{-19}$ J.

3 Particles ψ and ψ'

The e^+e^- collision ring SPEAR (SLAC) was of one of the two experiments (the other one being at BNL) which led to the discovery of ψ (now called J/ψ) particle, and thus of charm quark, announced on November 11th 1974. Indeed, ψ and ψ' can be produced through the reactions

$$e^+ \ e^- \to \psi \tag{3}$$

and

$$e^+ \ e^- \to \psi' \, . \tag{4}$$

The experimental configuration is such that the two beams collide head-on, and carry the same energy.

1. What should be the kinetic energies T_e of e^+ and e^- to allow for these two reactions? Show that the speed of electron is close to the speed of light, with a relative precision better than 10^{-6}.

2. The particle ψ' can decay at rest according to the reaction

$$\psi' \to \psi \ n\pi^+ \ n\pi^- . \tag{5}$$

What is the maximal value of n, the number of pairs of charged pions? In the following, the work in the rest frame of the ψ', and we consider the case $n = 1$.

3. What are the kinetic energies T_+ and T_- of the π^+ and π^- mesons when the particle ψ of the process (5) is produced at rest?

4. When ψ is not at rest, provide a relationship between T_+, T_-, the kinetic energy T of the particle ψ and the relative angle θ between the flight directions of the two pions.

Data: $m_e c^2 = 0.511$ MeV, $m_\psi c^2 = 3097$ MeV, $m_{\psi'} c^2 = 3686$ MeV, $m_{\pi^\pm} c^2 = 139.57$ MeV.

4 Maximum energy carried by a particle

We consider either the decay process

$$A \to D_1 + D_2 + D_3 \cdots D_N \tag{6}$$

or the scattering process

$$A + B \to D_1 + D_2 + D_3 \cdots D_N \, . \tag{7}$$

We first work in the center-of-mass frame R, denoting the total energy as E. Our aim is to compute the maximal energy $E_i(max)$ of particle i in this frame. To simplify notations, we denote $p_i = |\vec{p}_i|$. Without loss of generality, we first consider particle 1. We will come back to the general case in question 8.

1. Express $\vec{p}_1$ and E_1 in terms of $\vec{p}_2, \cdots \vec{p}_N$ and $E_2, \cdots E_N$ respectively.

2. We denote $E_r = E_2 + \cdots E_N$. Show that p_1 is maximal when E_r is minimal.

3. We now consider the center-of-mass frame R' of the $N-1$ particles $D_2, \cdots D_N$. Introducing the Lorentz factor γ between the two frames, relate E_r and E'_r, the energy of the $N-1$ particles respectively in frame R and frame R'.

Deduce that E_r is minimal when E'_r is itself minimal, and that this implies that each particle should be at rest in frame R'.

4. Explain why in such a configuration, every particle is flying in the same direction in frame R.

5. Show then that in this configuration,

$$\forall i \in \{2, \cdots N\}, \frac{\vec{p_i}}{m_i} = \frac{\vec{p_2}}{m_2}. \tag{8}$$

6. Prove that

$$p_1(max) = p_2 \frac{M}{m_2}, \tag{9}$$

with $M = m_2 + \cdots m_N$.

7. Show that

$$E = \sqrt{p_1(max)^2c^2 + m_1^2c^4} + \sqrt{p_1(max)^2c^2 + M^2c^4} \tag{10}$$

8. Derive finally the desired result, after some algebra and an obvious relabeling:

$$cp_i(max) = \frac{\sqrt{(E + m_ic^2 + M_ic^2)(E - m_ic^2 - M_ic^2)(E + m_ic^2 - M_ic^2)(E - m_ic^2 + M_ic^2)}}{2E} \tag{11}$$

with $M_i = m_1 + m_2 + \cdots m_{i-1} + m_{i+1} + \cdots m_N$.

5 A passing stick

A stick of length L moves past you at speed v. There is a time interval between the front end coinciding with you and the back end coinciding with you.

1. What is this time interval in your frame? Obtain the result by working in your frame.

2. Same question, but now working in the stick's frame. You should of course get the same result as in 1.!

3(***!). What is this time interval in the stick's frame? Obtain the result by working in your frame.

4. Same question, but now working in the stick's frame.